Unit - V

Sliding Contact Bearings

Lubrication is the science of reducing friction, by application of a suitable substance called lubricant, between the rubbing surfaces of bodies having relative motion.

Lubricants
- Liquid lubricants, like mineral or vegetable oil
- Semi-solid lubricants, like grease
- Solid lubricants, like graphite or Molybdenum disulphide.

Function of lubricants :-

(1) reduce friction
(2) reduce or prevent wear
(3) carry away heat generated due to friction.
(4) Protect journal & bearing from corrosion.

modes of lubricants :-

→ **Zero Film lubrication** :- lubrication which operates without any lubricant.

→ **Thick film lubricant** : It is a condition of lubrication when two surfaces of bearing in relative motion are completely separated by film or fluid.

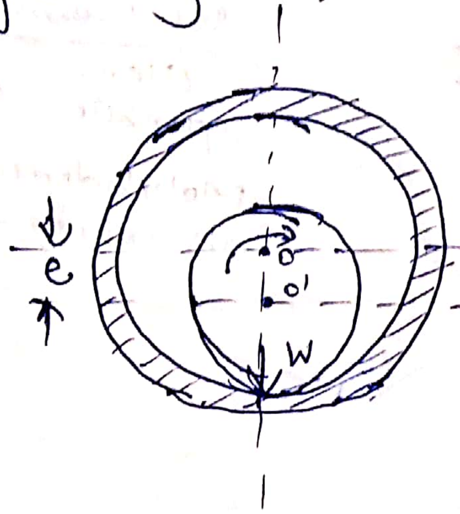
The resistance to motion is due to viscous resistance of fluid, hence viscosity affect performance of bearing.

Thick film lubrication

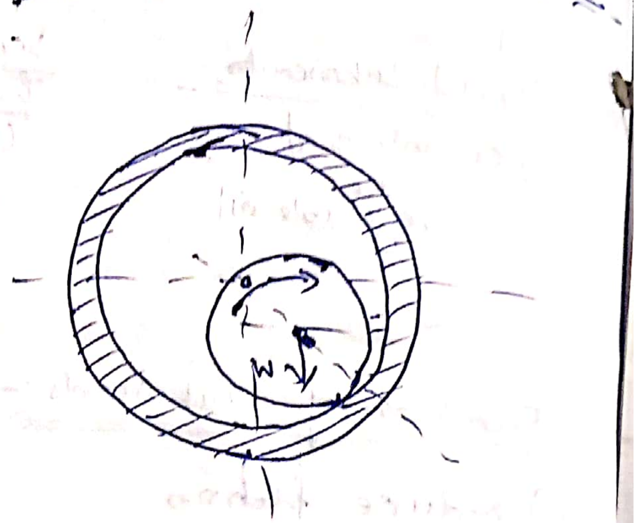
Hydrodynamic lubrication

It is defined as a system of lubrication in which load supporting fluid film is created by shape & relative motion of sliding surfaces

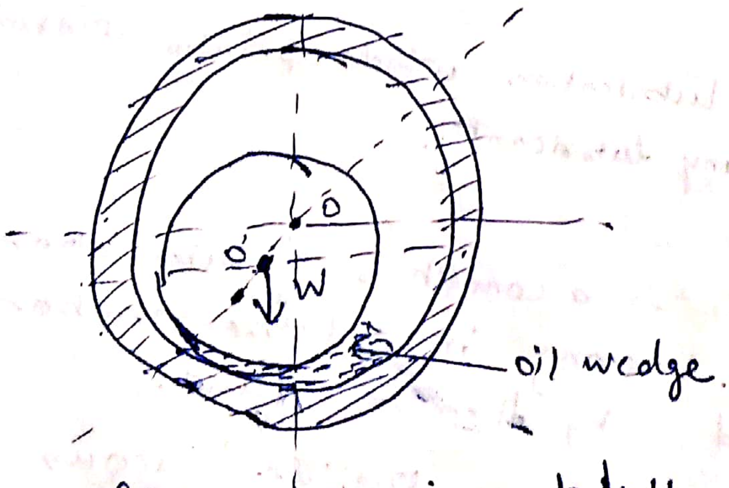
Hydrostatic lubrication.



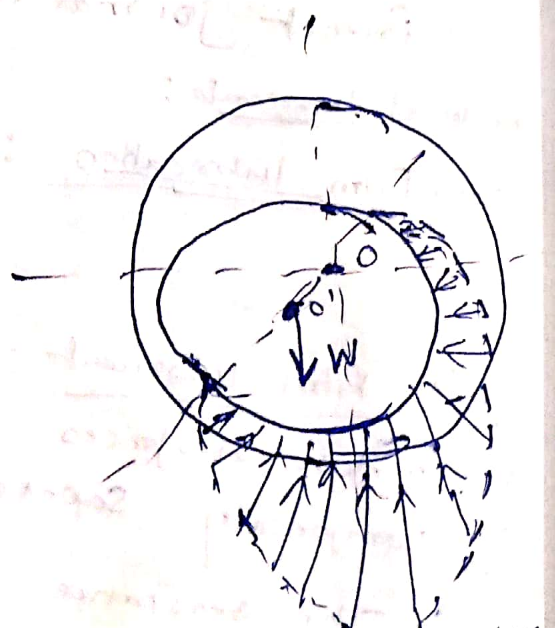
(a) Initial shaft is at rest & sink at Bottom.



(b) Shaft climbs the bearing surface as it is started.



(C) Shaft is at full speed the wedge is formed below.



Pressure distribution in hydrodynamic lubrication.

Since pressure is created within system due to rotation of shaft, this type of bearing is known as self acting bearing.

The pressure generated supports external load (W)

~~The~~ o The lubricants should be continously supplied, no need of maintaing pressure.

This lubrication is used in bearings mounted on engines & centrifugal pumps.

* Journal bearing is a sliding contact bearing working on hydrodynamic lubrication & which support load in radial direction.

Journal Bearing

Full journal

- angle of contact of bushing with journal is 360°
- They can take load in any radial direction.
- mostly used in industrial application.

Partial journal.

- angle of contact is ~~less~~ than 180° mostly 120°.
- They can take load only in one radial direction
- used in ~~rail road~~ railroad - cars.
- They are simple in construction in compare to full
- Easy to supply lubricating oil.
- Friction loss is less ~~that~~ therefore temp. rise is low.

There are two terms related partial & full bearings i.e.

① clearance Bearing ② Fitted Bearing.

① clearance ~~Bea~~ Bearing:-

The radius of journal is less than the radius of bearing. Most of the journal Bearings are of this type.

② Fitted Bearing:

The radius of journal & Bearings are equal. Fitted bearing must be partial bearing. & the journal must run eccentric w.r.t bearing to provide space for lubricating oil.

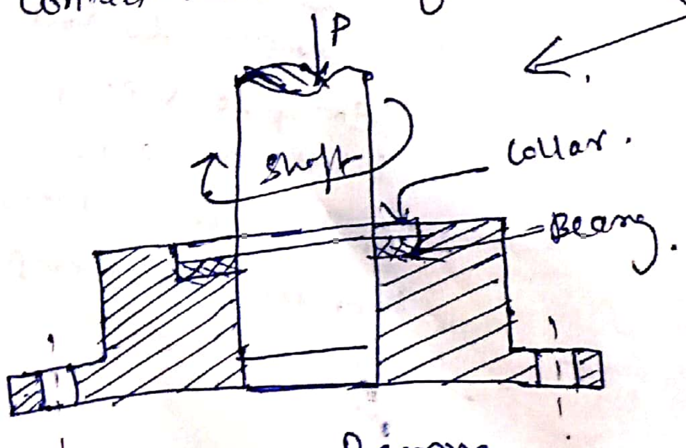
Thrust Bearings

Axial Bearing or Footstep Bearing

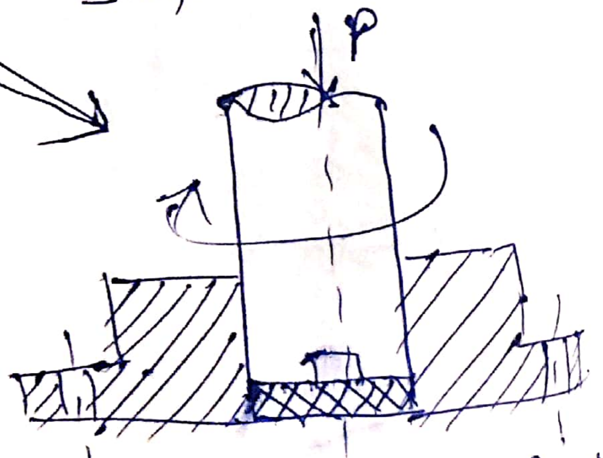
→ It is a thrust bearing in which the end of shaft is contact with bearing surface.

Collar Bearing.

→ It is a thrust Bearing in which a collar integral with the shaft is in contact with Bearing surface.



Collar Bearing

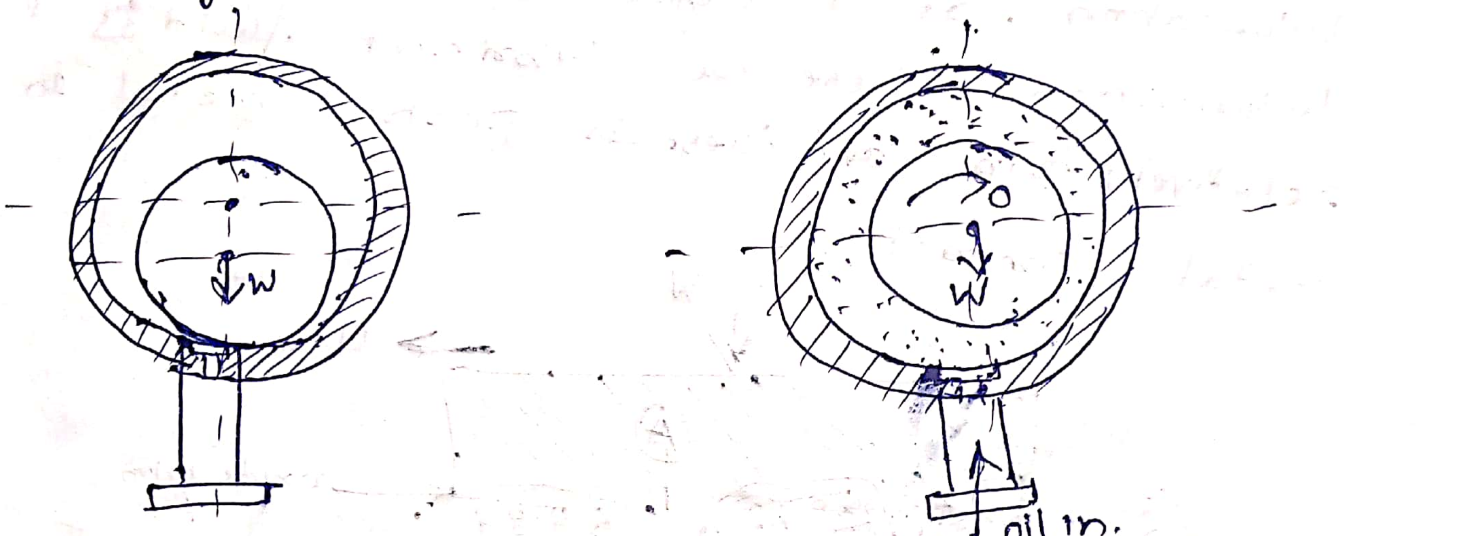


Footstep Bearing

Hydrostatic lubrication

It is defined as system of lubrication in which the load supporting fluid film, separating the two surfaces is created by external ~~sourde~~ source like a pump, supplying sufficient fluid under pressure.

This is also called externally pressured Bearing.



(a) Journal at rest

(b)

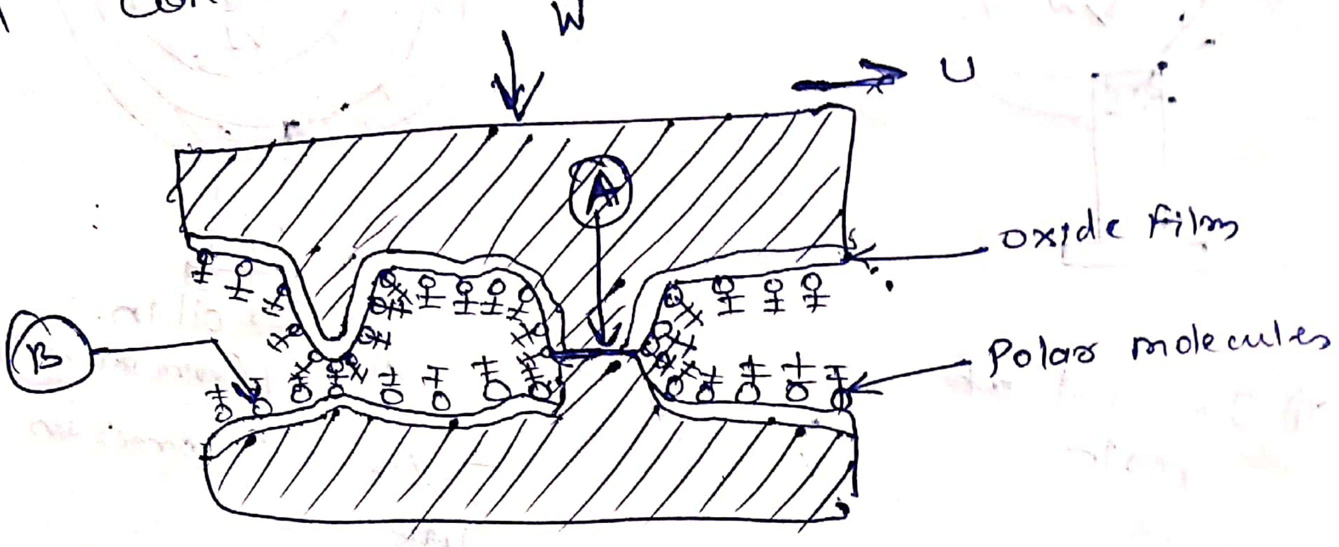
- As oil comes in the shaft lifts.
- They are used on vertical turbo generators, centrifuges and ball mills.

Comparission

Hydrostatic lubrication	Hydrodynamic lubrication
(1) high load carrying capacity even at low speed.	(1) simple in construction, easy to maintain
(2) No starting friction.	(2) lower in initial & maintain cost.
(3) no rubbing action at any operating speed.	

Thin Film lubrication :-

It is also called as boundary lubrication. It is defined as a condition of lubrication where the lubricant fluid is relatively thin and there is partial metal to metal contact.



(A) → Metal to metal Contact

(B) → cluster of molecules

molecules in which there is permanent Seperation of positive and negative charges are Called polar Molecules.

Region B [molecules cohering to ~~one~~ one another and adhering to the surface, adhering to the surface to form a compact film which prevent metal to metal contact. This is Partial lubrication]

Region A — Metal to metal contact takes place, junctions are formed at high spot and shearing takes place due to relative motion

Performance of Bearing Boundary depends upon
- ① chemical Composition (Region B)
- ② Surface Roughness (Region A)

The Hydrodynamic bearing also operates under the boundary lubrication when the speed is very low or load is excessive.

* Elastohydrodynamic Lubrication:-

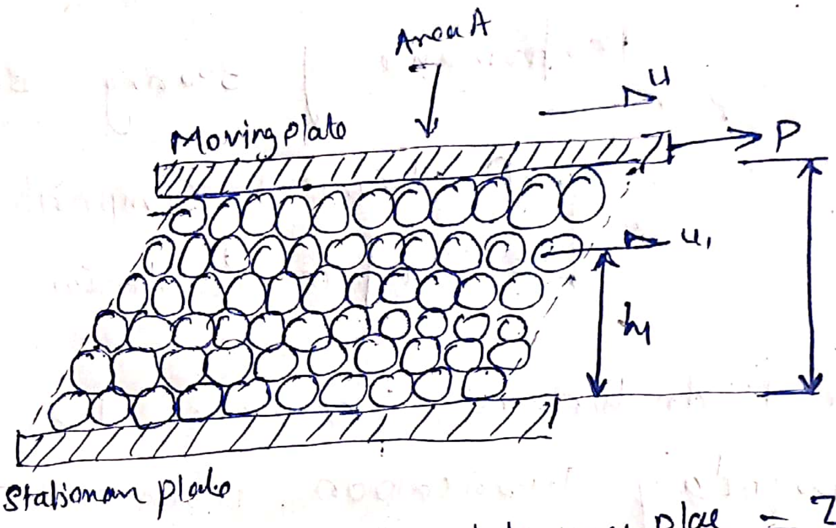
When the fluid film pressure is high and the surfaces to be seperated are not sufficiently rigid, there is elastic deformation of the contacting surfaces.

The hydrodynamic film is developed due to elastic deflection of the parts, this mode of lubrication is called ~~elasto~~ elastohydrodynamic lubrication.

It occurs in gears, cams, rolling contact bearings.

* Viscosity

Viscosity is defined as the internal frictional resistance offered by a fluid to change its shape or relative motion of its parts.



Molecules in contact with stationary plate = Zero velocity

— " — " — " moving — " = U velocity

— " — " — " Intermediate layers with,

velocity $\propto$ distance from stationary plate

$\therefore U \propto h$.

then, $U_1 \propto h$.

So, $\boxed{\frac{U}{h} = \frac{U_1}{h_1} = \frac{U_2}{h_2}}$ — This type of orderly movement is called streamline laminar or viscous flow

So,

$$\text{Shear stress} = \frac{\text{Tangential Force}}{\text{Shear Area}} = \frac{P}{A}$$

$$\text{Rate of shear} = \frac{U}{h}$$

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Newton's law of ~~...~~ viscosity,

Shear stress $\propto$ rate of shear at any point.

$$\left(\frac{P}{A}\right) \propto \left(\frac{U}{h}\right)$$

$$\frac{P}{A} = \mu\left(\frac{u}{h}\right)$$

$$P = \mu A\left(\frac{u}{h}\right)$$

when velocity distribution is non-linear with respect to h

$$\boxed{P = \mu \cdot A\left(\frac{du}{dh}\right)}$$ — u, non linear w.r.t h.

μ = ~~...~~ absolute viscosity.

$$\mu = \frac{Ph}{AU}$$

$$= \frac{(N)(mm)}{(mm^2)(mm/sec)}$$

$$= \frac{N-sec}{mm^2} \quad \text{or} \quad Mpa-Sec.$$

Popular unit is Poise $= \frac{dyne-sec}{cm^2}$

Viscosity of most of lubricating oils are in Centi-Poise (cP)

$$cP = \frac{Poise}{100}.$$

So, two notations are used,

μ = viscosity in Units (N-s/mm²) or (Mpa-sec)

Z = — " — " — " — (cP)

$$1\ cP = \frac{1}{10^2}\cdot Poise$$

$$= \frac{1}{10^2}\cdot\frac{dyne-sec}{cm^2}.$$

$$= \frac{1}{10^2}\left(\frac{N}{10^5}\right)\left(\frac{S}{10^2\,mm^2}\right)$$

$$= (10^{-9})\ N\text{-}S/mm^2$$

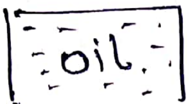
$$1\ N\text{-}S/mm^2 = 1\ Mpa\cdot S = (10^9)\ cP$$

$$\mu = \frac{Z}{10^9}.$$

* Measurement of Viscosity :-

→ difficult to measure viscosity with two parallel plates.

→ Popular methods – Time required for ~~standard~~ a given volume of oil to pass through a capillary tube of standard dimensions.



← Constt temp. bath

Reservior.

methods for measuring viscosity

① Saybolt viscometer (USA)

60 cm^3 oil is Passed through capillary of std. dimensions. & time is measured.

$$Z_K = \left[0.22t - \frac{180}{t}\right]$$

t = viscosity in Saybolt Universal second.

Kinematic viscosity $Z_K = \frac{Z}{\rho} = \frac{\text{absolute visc.}}{\text{density}}$.

② Redwood viscometer (UK)

volume used here is 50 cm^3 and time measured in Redwood seconds.

③ Engler Viscometer (Indian Subcont.)

viscosity is measured in terms of Engler degree (°E)

$$°E = \frac{\text{time taken by oil}}{\text{time taken by water}}$$

at same temp.

* Viscosity Index

Viscosity in oil is due to intermolecular forces

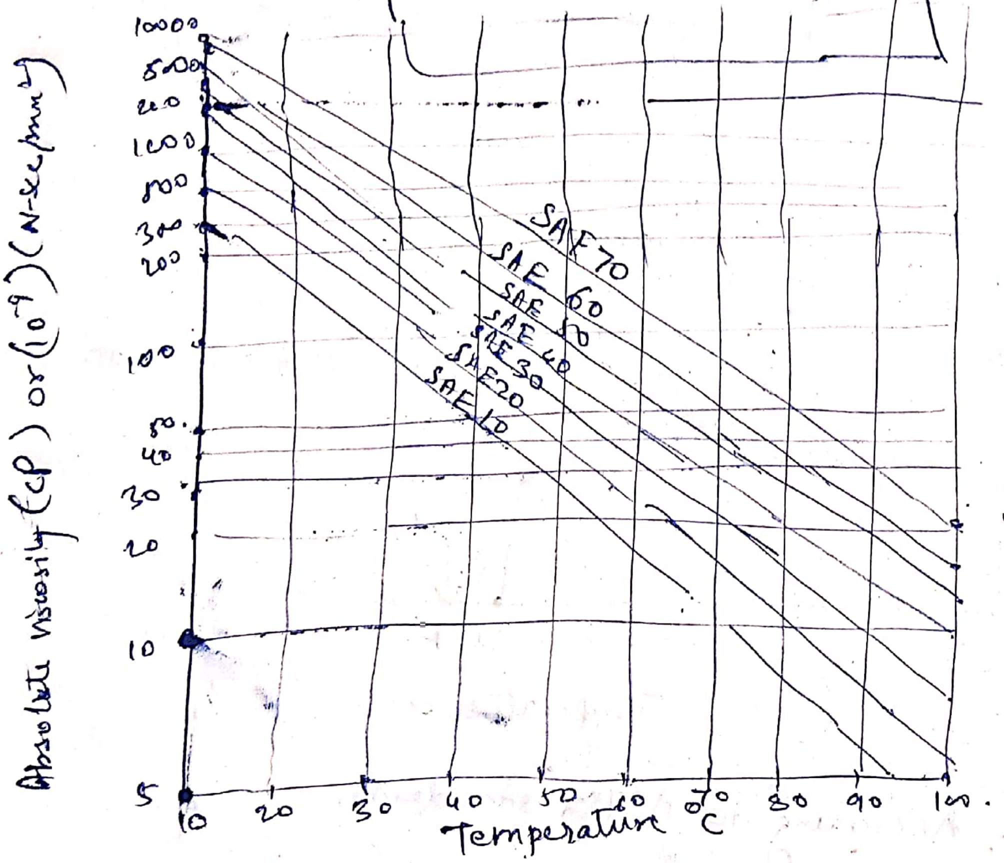
temp. ↑ then, viscosity ↓ as intermolecular forces ↓

Relation,

$$\boxed{\log \mu = A + \frac{B}{T}}$$

where A & B are constants.

T = Absolute temp.



Rate of change of viscosity with temp. is indicated by numbers of viscosity Index

The viscosity Index is defined as an arbitrary number used to characterize the variation of the kinematic viscosity of ~~lub~~ Lubricating oil with the temperature.

Two group of oil.

Reference 1

VI is 100

very small change with temperature

Reference 2.

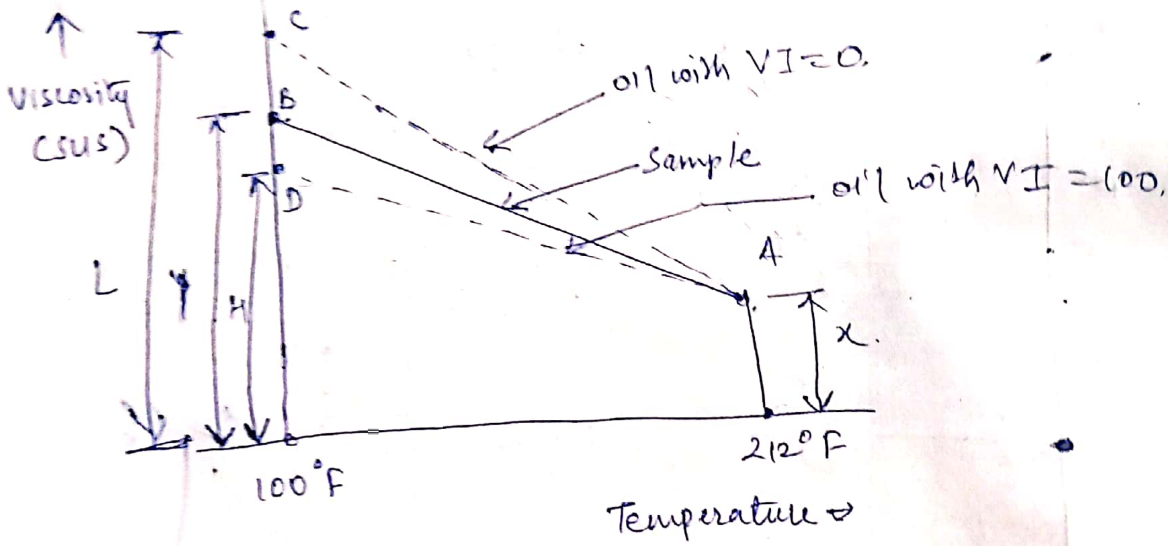
VI is 0

very large change of viscosity with temp.

To measure viscosity Index, the procedure followed is

① Measure given sample oil viscosity at 100°F and 212°F.

Sr. No	Temp.	Viscosity	Points in graph
1	100°F	y	B
2	212°F	x	A



So, According to ASTM standards,

$$VI = \left(\frac{L-Y}{L-H}\right) \times 100\%$$

An oil with VI=70 has less rate of change of viscosity with temperature compared with an oil with VI=60.

* Petroff's Equation :-

Petroff's equation is used to determine the coeff. of friction of journal bearing.

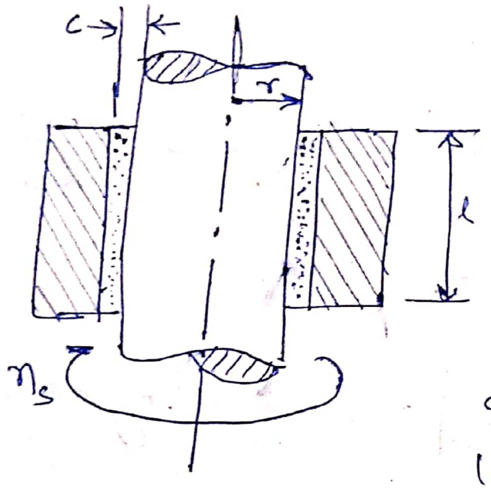
Assumptions :-

① Shaft is Concentric with the bearing.

② Bearing is Subjected to light load.

In practice such condition not occurs

Petroff's equation defines the group of dimensionless parameters that govern the frictional properties of bearing.



n_s = journal speed (rev/sec)

c = clearance (mm)

l = length of bearing (mm)

r = radius of journal (mm)

velocity of the surface of journal,

$$U = (2\pi r) n_s \quad \text{— (1)}$$

and we know, newton's law of viscosity

$$P = \mu A \left(\frac{U}{h}\right) \quad \text{— (2)}$$

P = tangential frictional force.

A = Area of journal surface $= (2\pi r)\, l$ — (3)

U = Surface velocity $= 2\pi r\, n_s$. — (4)

h = Clearance c.

Putting equtn ~~(1) & in (2)~~ (1), (3), (4) in (2)

~~$R = \mu A$~~

$$P = \mu \cdot (2\pi r) l \cdot \frac{(2\pi r) n_s}{c}$$

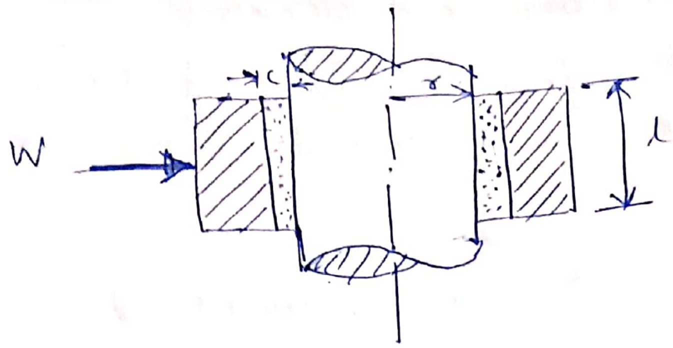
$$\boxed{P = \frac{4\pi^2 r^2 l \mu \cdot n_s}{c}}$$ — tangential frictional force

Tangential frictional torque,

$$(M_t)_f = P \cdot r$$

$$= \frac{4\pi^2 r^2 l \mu \cdot n_s}{c} \cdot r$$

$$\boxed{(M_t)_f = \frac{4\pi^2 r^3 l \cdot \mu \cdot n_s}{c}}$$



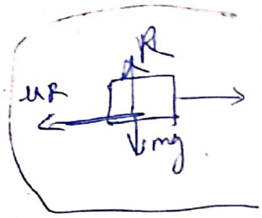
let us consider a load W acting on bearing.

So,

bearing pressure, $p = \frac{W}{\text{(Projected Area)}}$

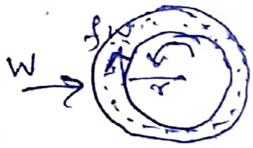
$$p = \frac{W}{2r \cdot l}$$

$$\boxed{W = p \cdot 2r \cdot l}$$ — load acting on bearing.



frictional torque,

$$(M_t)_f = (\text{frictional force}) \times r$$



$$(M_t)_f = (f \cdot W) \cdot r$$

$$(M_t)_f = f(2p \cdot r \cdot l) \cdot r$$

but,

$$\frac{\cancel{4}^{2}\pi^2 \cdot r^3 \cancel{l} \cdot \mu \cdot n_s}{c} = f \cdot \cancel{2}p \cdot r^2 \cdot \cancel{l}$$

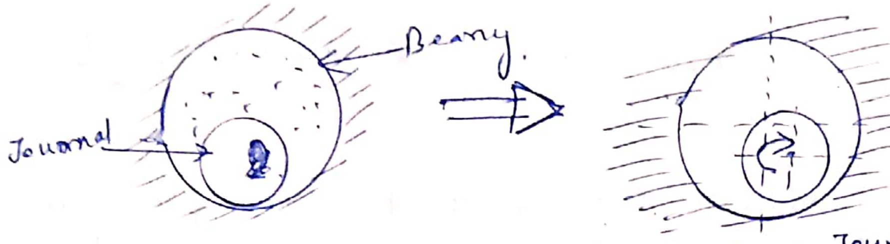
$$f = \frac{2\pi^2 r \cdot \mu \cdot n_s}{c \cdot p}$$

$$\boxed{f = (2\pi^2)\left(\frac{r}{c}\right)\left(\frac{\mu \cdot n_s}{p}\right)}$$

This is called Petroff's Equation

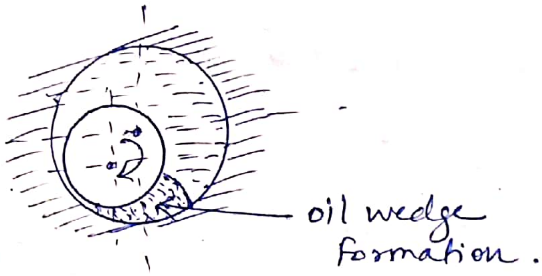
* McKee's Investigation :-

Hydrodynamic Lubrication

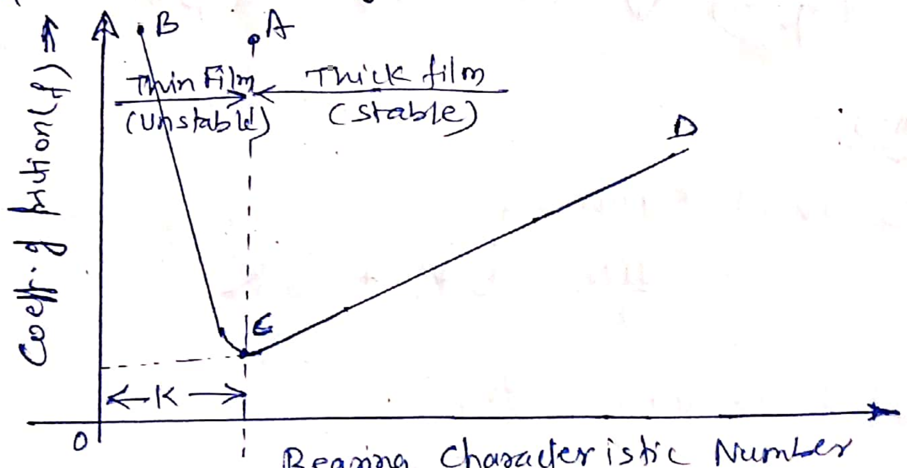


Initial State

As ~~Bearing~~ Journal starts rotating there is partial metal to metal contact & partial lubricant film.



As speed is increased, more and more lubricant is forced into the wedge~~d~~-shaped clearance space and sufficient pressure is built up, seperating the surface of the journal and the bearing. This is thick lubrication.



Bearing Characteristic Number $\frac{\mu N}{P} \rightarrow$

Transition from thin film lubrication to thick film ~~lubrication~~ hydrodynamic lubrication can be better visualized by $\mu N/P$ curve.

- This is experimental curve developed by McKee Brothers

- Bearing characteristic number is a dimensionless group of parameters.

Bearing characteristic number $= \left(\frac{\mu N}{p}\right)$

① Region BC → ① Partial metal to metal contact] thin film or Boundary lubrication.
② Partial patches of lubricant

② Region CD → ① There is relatively thick film of lubricant.
② Hydrodynamic lubrication takes place.

③ AC line ⇒ It is dividing line between two modes of lubrication.

④ Summarize :- $f \downarrow$ at C

The value of $\left(\frac{\mu N}{p}\right)$ at minimum value of f is called "bearing modulus" denoted by K.

$\left(\frac{\mu N}{p}\right) \downarrow$ when $N \downarrow$ and $p \uparrow$.

Guidelines:

① To avoid seizure,

$$\frac{\mu N}{p} = 5K \text{ to } 6K$$

② If fluctuating loads,

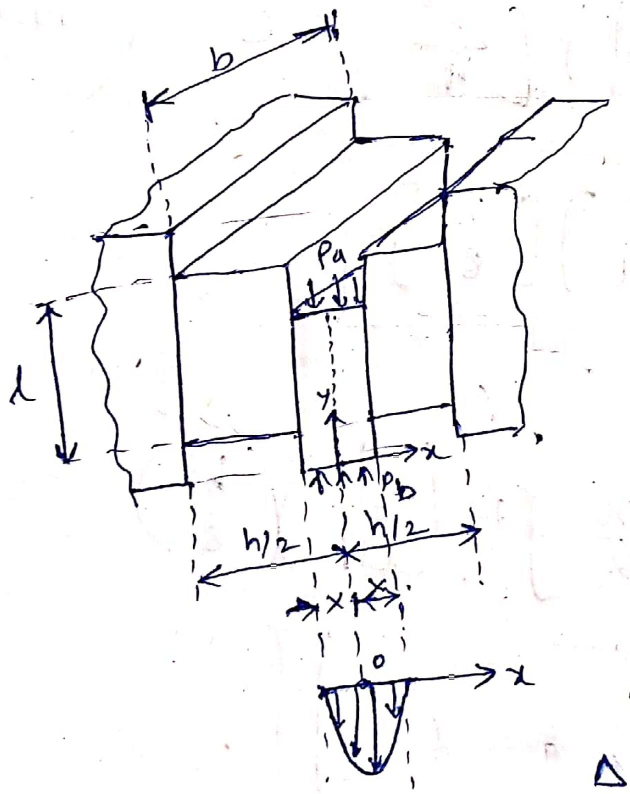
$$\frac{\mu N}{p} \geq 15K$$

when viscosity is very low, $\left(\frac{\mu N}{p}\right) \downarrow$ and boundary lubrication will result.

Viscosity of lubricant ↓ then, $\left(\frac{\mu N}{p}\right)$ ↓ ⇒ lubricant will not seperate the surface of journal & bearing. and hence metal to metal contact occur and wear takes place.

$\left(\frac{\mu N}{p}\right)$ curve is important because it defines the stability of hydrodynamic journal bearings and help to visualize the transition from boundary lubrication to thick film lubrication.

* Viscous Flow through Rectangular slot :-



Flow of lubricating oil through rectangular slot is shown in Fig.

The dimensions of b is very large compared to h. Hence losses at sides are neglected

The pressure difference between the central slice

$$\Delta p = p_a - p_b$$

Net force on slice

$$P = \Delta p \cdot (2x)(b)$$

This net force will be equal to the resistance on both surface of the slice due to viscosity of lubricant i.e.

$$P = \mu A \left(\frac{du}{dh}\right) \quad (2)$$

So, $\therefore A = 2lb$ — shear Area.

$$2xb\,\Delta p = -\mu(2lb)\left(\frac{dv}{dx}\right)$$

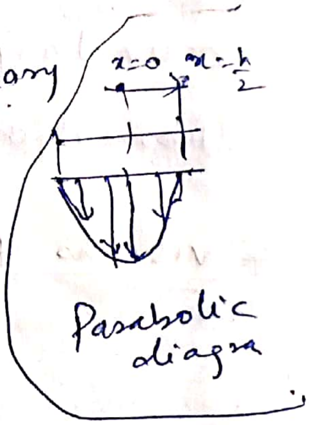
$$dv = -\left(\frac{\Delta p}{\mu l}\right) \cdot x \cdot dx.$$

$$\int dv = -\left(\frac{\Delta p}{\mu l}\right) \cdot \int x \cdot dx$$

$$v = -\left(\frac{\Delta p}{\mu l}\right)\left[\frac{x^2}{2}\right] + c$$

To find c value, using boundary conditions,

i.e when $x = \frac{h}{2}$, $v = 0$.



$$0 = -\left(\frac{\Delta p}{\mu l}\right)\left[\frac{h^2}{8}\right] + c$$

$$\boxed{c = \left(\frac{\Delta p}{\mu l}\right)\left[\frac{h^2}{8}\right]}$$

So,

$$v = -\left(\frac{\Delta p}{\mu l}\right)\left[\frac{x^2}{2}\right] + \left(\frac{\Delta p}{\mu l}\right)\left(\frac{h^2}{8}\right)$$

$$\boxed{v = \frac{\Delta p}{2\mu l}\left[\frac{h^2}{4} - x^2\right]}$$

But from parabolic diagram

when $x = 0$, $V = V_{max}$

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So,

$$V_{max} = \frac{\Delta p}{2\mu}\left[\frac{h^2}{4}\right]$$

$$\boxed{V_{max} = \frac{\Delta p \cdot h^2}{8\mu l}}$$

For parabolic profile

Average height $= \frac{2}{3}$ (max^m height)

So,

Average velocity $= \frac{2}{3}(V_{max})$

$$V_{avg} = \frac{\cancel{2}}{3}\left(\frac{\Delta p \cdot h^2}{\cancel{8}_4 \mu l}\right)$$

$$\boxed{V_{avg} = \frac{\Delta p\, h^2}{12\mu l}}$$

So, lubricant flow Q

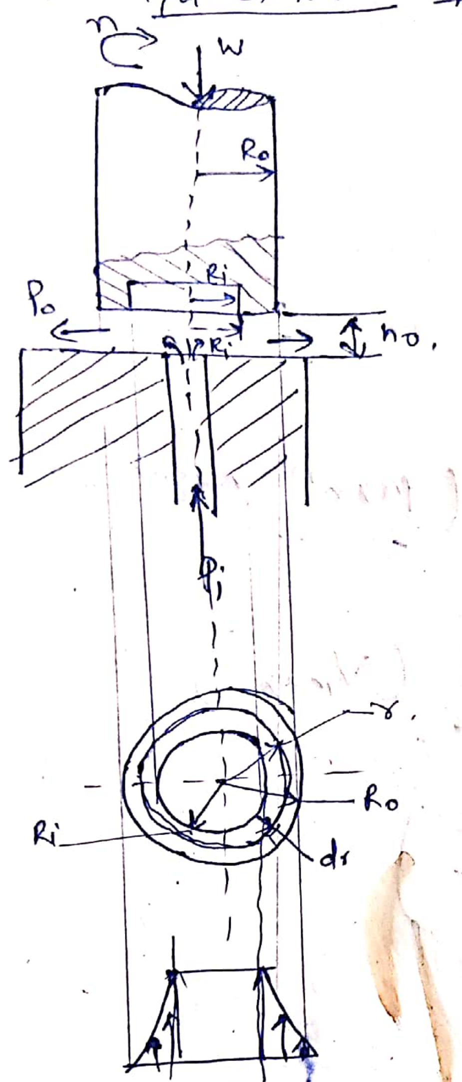
$$Q = (V_{avg}) \times (\text{area})$$

$$= \frac{\Delta p h^2}{12\mu l} \times bh$$

$$\boxed{Q = \frac{\Delta p h^3 \cdot b}{12\mu l}}$$

This is fundamental equation for viscous flow through Rectangular slot.

Hydrostatic Step Bearing



W = thrust load

Ro = outter radius of shaft

Ri = inner radius of recess

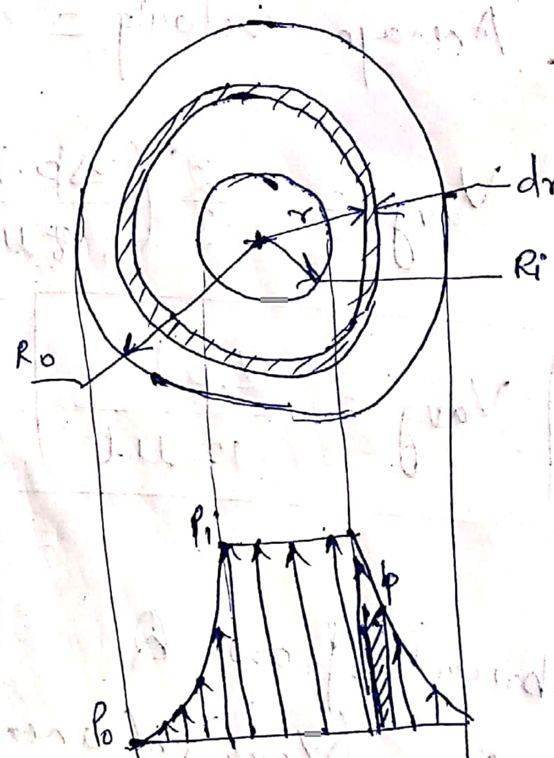
Pi = supply of inlet pressure

Po = outlet or atmospheric pressure

ho = fluid film thickness (mm)

Q = flow of the lubricant mm³/sec

μ = viscosity of the lubricant (MPa-s)



The flow of the lubricant through elemental ring is given by

$$Q = \frac{\Delta p\, b h^3}{12 \mu l}$$

In this case, $l = dr$

and, $b = 2\pi r$

$h = h_o$

$\Delta p = dp$

So,

$$Q = -\left[\frac{dp \cdot 2\pi r \cdot h_o^3}{12 \cdot \mu \cdot dr}\right]$$

$$Q = -\left[\frac{\pi r h_o^3}{6\mu}\right] \cdot \frac{dp}{dr}$$

-ve sign indicates that pressure decreases as the radius increases

So,

$$dp = -\left[\frac{6\mu Q}{\pi h_o^3}\right]\frac{dr}{r}$$

Integrating

$$\int dp = -\left[\frac{6\mu Q}{\pi h_o^3}\right]\int \frac{dr}{r}$$

$$\boxed{p = -\left[\frac{6\mu Q}{\pi h_o^3}\right] \log_e r + C}$$

By Boundary Conditions,

$p = 0$ at $r = R_o$

$$0 = -\left[\frac{6\mu Q}{\pi h_o^3}\right] \log_e R_o + C$$

$$\boxed{C = \left[\frac{6\mu Q}{\pi h_o^3}\right] \log_e R_o}$$

So equtn becomes

$$p = \left[\frac{6\mu Q}{\pi h_o^3}\right]\left[\log_e(R_o) - \log_e(r)\right]$$

$$\boxed{p = \frac{6\mu Q}{\pi h_o^3}\left[\log_e\left(\frac{R_o}{r}\right)\right]}$$

Similarly Second Boundary Conditions.
i.e.

$$p = P_i \quad \text{when} \quad r = R_i$$

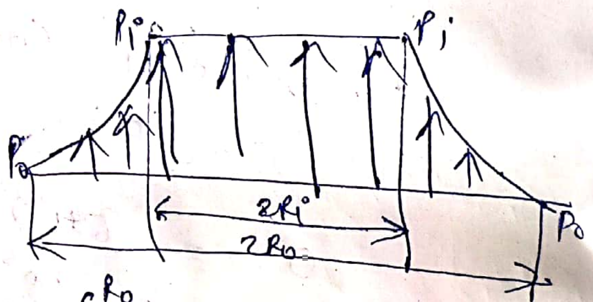
So,

$$P_i = \frac{6 \mu Q}{\pi h_0^3} \log_e \left[\frac{R_o}{R_i} \right]$$

$$\boxed{Q = \frac{\pi h_0^3 P_i}{6 \mu \log_e \left(\frac{R_o}{R_i} \right)}}$$

This is equation gives the flow requirement for bearing.

* Load Carrying Capacity of Bearing = [load supported at central recess area i.e P_i = Constt] + [Load supported at Annular Area i.e R_i to R_o]



Hence,

$$W = P_i (\pi R_i^2) + \int_{R_i}^{R_o} p \cdot 2\pi r \cdot dr$$

So substituting pressure equn $p = \frac{6 \mu Q}{\pi h_0^3} \left[\log_e \frac{R_o}{r} \right]$

$$W = \pi R_i^2 \cdot P_i + \int_{R_i}^{R_o} \frac{6 \mu Q}{\pi h_0^3} \left(\log_e \frac{R_o}{r} \right) \cdot 2\pi \cdot r \cdot dr$$

$$W = \pi R_i^2 P_i + \frac{12 \mu Q}{h_0^3} \int_{R_i}^{R_o} \log_e \left(\frac{R_o}{r} \right) \cdot r \cdot dr$$

So, Integration of $\int_{R_i}^{R_o} \log_e \left(\frac{R_o}{r} \right) r \cdot dr$

let $\boxed{U = \log_e\left(\frac{R_o}{r}\right)}$

$$\frac{du}{dr} = \frac{1}{(R_o/r)} \cdot \left(-\frac{R_o}{r^2}\right)$$

$$\frac{du}{dr} = \left(\frac{r}{R_o}\right)\left(-\frac{R_o}{r^2}\right)$$

$$\frac{du}{dr} = -\frac{1}{r}.$$

$$\boxed{du = -\frac{1}{r} \cdot dr.}$$

and let $\boxed{r \cdot dr = dv.}$

$$\int r \cdot dr = \int dv$$

$$\boxed{v = \frac{r^2}{2}.}$$

So, substituting back in equation

$$= \int_{R_1}^{R_o} \log_e\left(\frac{R_o}{r}\right) \cdot r \cdot dr$$

$$= \int \underset{I}{u} \, \underset{II}{(1)} dv.$$

No Integrating by Parts

$$= \cancel{u \cdot \int (1) dv - \int du \int dv \, dv}.$$

$$= u \cdot \int (1) \cdot dv - \int \frac{du}{du} \cdot \left(\int (1) \cdot dv\right) \cdot du$$

$$= u \cdot v - \int \frac{du}{du} \cdot (v) \cdot du$$

$$= u \cdot v - \int v \cdot du.$$

$$= \left[\log_e\left(\frac{R_o}{r}\right) \cdot \frac{r^2}{2}\right] - \int \frac{r^2}{2} \cdot \left(-\frac{1}{r} dr\right)$$

$$= \frac{r^2}{2} \log_e\left(\frac{R_o}{r}\right) + \int \frac{r}{2} \cdot dr$$

$$= \left[\frac{r^2}{2} \log_e\left(\frac{R_o}{r}\right) + \left[\frac{r^2}{4}\right]\right]_{R_1}^{R_o}$$

$= \int uv \, dx$

$= u \int v \, dx - \int u' \left(\int v \, dx\right) dx$

$$= \left[\left\{ \frac{R_o^2}{2} \log_e\left(\frac{R_o}{R_o}\right) + \frac{R_o^2}{4} \right\} - \left\{ \frac{R_i^2}{2} \log_e\left(\frac{R_o}{R_i}\right) + \frac{R_i^2}{4} \right\} \right]$$

$$= \left[\left\{ \frac{R_o^2}{2} (0) + \frac{R_o^2}{4} \right\} - \left\{ \frac{R_i^2}{2} \log\left(\frac{R_o}{R_i}\right) \right\} - \frac{R_i^2}{4} \right]$$

$$= \frac{R_o^2 - R_i^2}{4} - \left(\frac{R_i^2}{2}\right) \log_e\left(\frac{R_o}{R_i}\right)$$

So,

$$W = \pi R_i^2 P_i + \frac{12\mu Q}{h_o^3} \left[\frac{R_o^2 - R_i^2}{4} - \left(\frac{R_i^2}{2}\right) \log_e\left(\frac{R_o}{R_i}\right) \right]$$

But $Q = \dfrac{\pi h_o^3 P_i}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}$

So,

$$W = \pi R_i^2 P_i + \frac{\cancel{12}^{2} \cancel{\mu}}{\cancel{h_o^3}} \left(\frac{\pi \cancel{h_o^3} \cdot P_i}{\cancel{6\mu} \log_e\left(\frac{R_o}{R_i}\right)} \right) \left[\frac{R_o^2 - R_i^2}{4} - \frac{(R_i)^2}{2} \log_e\left(\frac{R_o}{R_i}\right) \right]$$

$\cancel{W = \pi R_i^2 P_i}$

$$W = \pi P_i \left[R_i^2 + \frac{\cancel{2}}{\log_e\left(\frac{R_o}{R_i}\right)} \times \left(\frac{R_o^2 - R_i^2}{\cancel{4}_2} \right) - \frac{\cancel{2}}{\cancel{\log_e\left(\frac{R_o}{R_i}\right)}} \cdot \frac{(R_i)^2}{\cancel{2}} \cancel{\log_e\left(\frac{R_o}{R_i}\right)} \right]$$

$$W = \pi P_i \left[\cancel{R_i^2} + \frac{R_o^2 - R_i^2}{2 \log_e\left(\frac{R_o}{R_i}\right)} - \cancel{R_i^2} \right]$$

$$\boxed{W = \frac{\pi P_i}{2} \left[\frac{R_o^2 - R_i^2}{\log_e\left(\frac{R_o}{R_i}\right)} \right]}$$

Total Energy loss in hydrostatic Bearing is due to two Factors

(1) Energy required to pump lubricating oil (E_p)

$$E_p = Q(P_i - P_o)$$

$$= \frac{mm^3}{sec} \cdot \frac{N}{mm^2}$$

$$= \frac{N\text{-}mm}{sec}$$

or

$$E_p = Q(P_i - P_o) \times 10^{-3}$$

$\frac{N\text{-}m}{sec}$ or W.

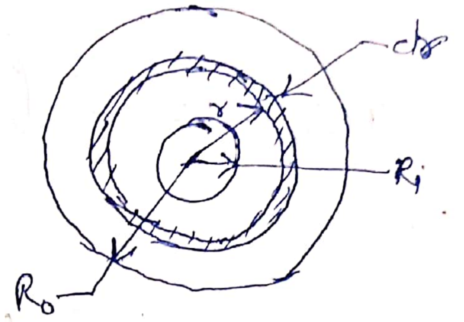
Therefore in

$(\cancel{kW})_p = Q(P_i - P_o) \times 10^{-6}$

P_p kW

$$\boxed{P_p = Q(P_i - P_o) \times 10^{-6} \ (kW)}$$

(2) Energy loss due to viscous friction.



The viscous resistance of ring is dF. This is determined by Newton's law of viscosity

$$dF = \mu A \left(\frac{U}{h}\right)$$

where,

$A = 2\pi r \cdot dr$

$U = r\omega$

$U = r \cdot \frac{2\pi N}{60}$

$h = h_o$

So,

$$dF = (\mu)(2\pi r \cdot dr)\left(\frac{r\omega}{h_o}\right)$$

$$dF = (\mu)(2\pi r)dr\left(\frac{2\pi N r}{(60)(h_o)}\right)$$

$$dF = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu N}{h_o}\right) \cdot r^2 \cdot dr$$

So the frictional torque is given by,

$$d(M_t)_f = dF \times r.$$

$$= \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu N}{h_o}\right) r^3 \cdot dr.$$

$$\int d(M_t)_f = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu N}{h_o}\right)\int_{R_i}^{R_o} r^3 . dr$$

$$(M_t)_f = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu N}{h_o}\right)\left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu N}{h_o}\right)\left[\frac{R_o^4}{4} - \frac{R_i^4}{4}\right]$$

$$\boxed{(M_t)_f = \frac{\pi^2}{60}\left(\frac{\mu N}{h_o}\right)\left[R_o^4 - R_i^4\right]} \quad \text{—(N-mm)}$$

$$\text{Power}_f = \frac{2\pi N (M_t)_f}{60 \times 10^6} \quad (\text{KW})$$

$$P_f = \frac{2\pi N}{60\times10^6}\left(\frac{\pi^2}{60}\right)\frac{\mu N (R_o^4 - R_i^4)}{h_o}.$$

$$\boxed{P_f = \left(\frac{1}{58.05\times10^6}\right)\frac{\mu N^2 (R_o^4 - R_i^4)}{h_o}}$$

So ~~to~~total power loss (KW). is

$$\boxed{P = P_p + P_f.} \quad (\text{KW})$$

16) Problem

Given:-

W = 500 KN

N = 720 rpm

D_o = 500 mm

D_i = 300 mm

h_o = 0.15 mm.

viscosity = 160 SUS

ρ = 0.86

To find:-

1. P_i = ?
2. Q = ?
3. P_p = ?
4. P_f = ?

$$\therefore W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log_e\left(\frac{R_o}{R_i}\right)}\right]$$

$$500\times10^3 = \frac{\pi(P_i)}{2}\left[\frac{(0.250)^2 - (0.150)^2}{\log_e\left(\frac{0.250}{0.150}\right)}\right]$$

$$\boxed{P_i = 4.065 \ N/mm^2}$$

(2) Saybolt Universal Seconds (SUS)

Kinematic viscosity,

$$Z_k = \left[0.22t - \frac{180}{t}\right]$$

$$= \left[0.022(160) - \frac{180}{160}\right]$$

$$\boxed{Z_k = 34.075 \ cSt}$$ — Centi Stokes

$$Z_k = \frac{Z}{\rho}$$

$$34.075 = \frac{Z}{0.86}$$

Absolute viscosity

$$\boxed{Z = 29.3 \ cP}$$

$$\mu = \frac{Z}{10^9}$$

$$\boxed{\mu = 29.3\times10^{-9} \ N\text{-}s/mm^2}$$

$$Q = \frac{\pi h_o^3 P_i}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{\pi(0.15)^3(4.065)}{6(29.3\times10^{-9})\log_e\left(\frac{250}{150}\right)}$$

$Q = 0.48 \times 10^6 \ mm^3/sec.$

$= 0.48 \times 10^6 \times 10^{-3} \ cc/sec.$ $\quad (\because 10^3 cc = 1 \ litre)$

$= 0.48 \times 10^6 \times 10^{-3} \times 10^{-3}$ litre/sec.

$= 0.48$ litre/sec

$= \cancel{0.48} \ 0.48 \times 60$ lit/min

$$\boxed{Q = 28.8 \text{ litre/min}}$$

(3) Power loss in pumping (P_p)

$P_p = Q(P_i - P_o) \times 10^{-6}$ — kW.

$= 0.48 \times 10^6 (4.065 - 0)(10^{-6})$

$$\boxed{P_p = 1.95 \text{ kW.}}$$

(4) Power loss in friction (P_f)

$$P_f = \left(\frac{1}{58.05 \times 10^6}\right) \frac{\mu N^2 (R_o^4 - R_i^4)}{h_o}$$

$$P_f = \frac{1}{(58.05 \times 10^6)} \cdot \frac{(29.3 \times 10^{-9})(720)^2 [(200^4 - 100^4)]}{0.15}$$

$$\boxed{P_f = 5.93 \text{ kW}}$$

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16.2

Problem

$W = 500$ KN

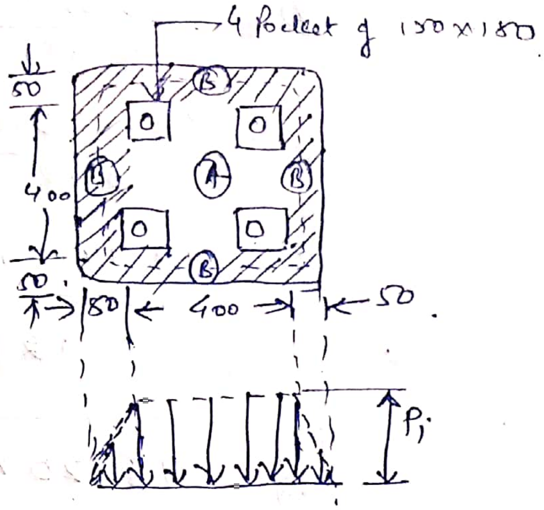
$h_o = 0.15$ mm

$Z = 250$ cP

$Z = 250 \times 10^{-9}$ N-s/mm²

Pressure in Area A is Uniform

Pressure in Area B is linear



The area of B can be assumed as ~~straighting~~ straightened out and has length equal to the mean length shown by dotted line.

Find:-

(1) $P_i = ?$

(2) $Q = ?$ (Litre/min)

Ans:-

(1) From pressure diagram we can say that.

Average Pressure in Area A = P_i

Average " " " B = $\frac{1}{2} P_i$

So,

$$W = P_i\,(\text{Area of } A) + \frac{1}{2} P_i\,(\text{Area of } B)$$

$$500 \times 10^3 = P_i\left[(400 \times 400) + (0.5)\{(500 \times 500) - (400 \times 400)\}\right]$$

$$500 \times 10^3 = P_i\left[160000 + 0.5(250000 - 160000)\right]$$

$$500 \times 10^3 = P_i\,(160000 + 45000)$$

$$500 \times 10^3 = P_i\,(205000)$$

$$P_i = \frac{500}{205}$$

$$\boxed{P_i = 2.44 \text{ N/mm}^2}$$

2) For flow requirement we have straightened out Area B. So that its length equal to $450 \times 4 = 1800$ mm.

$$Q = \frac{\Delta p\, b h^3}{12 \mu l}$$

$$= \frac{(P_i - P_o)\, b \cdot h^3}{12 \mu l}$$

$$= \frac{(2.44 - 0)(1800)(0.15)^3}{12(250 \times 10^{-9})(50)}$$

$$\boxed{Q = 98820 \ mm^3/sec}$$

1cm = 10 mm
1mm = 10^{-1} cm

1 litre = 10^3 cc
~~10^{-3} litre = 1 cc~~
1cc = 10^{-3} litre

$$Q = 98820 \times 10^{-3} \ cm^3/sec$$

$$= 98820 \times 10^{-3} \times 10^{-3} \ \text{litre/sec}$$

$$= 98820 \times 10^{-6} \times 60 \ \text{litre/min}$$

$$\boxed{Q = 5.93 \ \text{litre/min}}$$

10.3 Problem: Given:

Flow over corners is neglected

$W = 100$ kN

$h_0 = 0.02$ mm

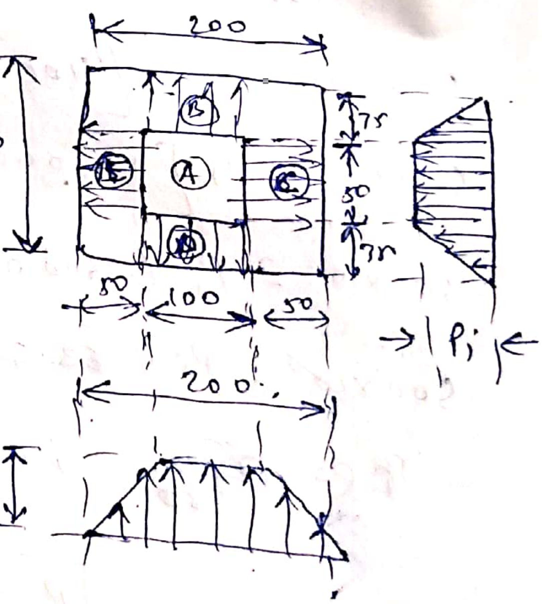
$\mu = 300$ cP

$= 300 \times 10^{-9}$ N-s/mm²

Find:-

① $P_i = ?$

② $Q = ?$



~~$W = P_i(\text{Area of } 2\times A) + P_i(\text{Area of B}$~~

$$W = P_i(\text{Area of A}) + \left[\frac{P_i}{2}(2 \times \text{Area B}) + \frac{P_i}{2}(2 \times \text{Area C})\right]$$

$$100 \times 10^3 = P_i\left[(50 \times 100) + \left\{\frac{1}{2}(2 \times 75 \times 100) + \frac{1}{2}(2 \times 50 \times 50)\right]\right.$$

$$100 \times 10^3 = P_i[15000]$$

$$P_i = \frac{100}{15}$$

$$\boxed{P_i = 6.666\ N/mm^2}$$

(2) $$Q = \frac{\Delta p\, bh^3}{12\mu l}$$

$$Q_B = \frac{(\cancel{P_i}^{6.66} - \cancel{P_o}^{0}) \times 100 \times (0.02)^3}{12 \times (300 \times 10^{-9})(75)}$$

$$\boxed{Q_B = 19.75\ mm^3/sec}$$

(3) $$Q_C = \frac{6.67 \times (50)(0.02)^3}{12 \times (300 \times 10^{-9})(50)}$$

$$\boxed{Q_C = 14.82\ mm^3/sec}$$

$$Q_{Total} = 2Q_B + 2Q_C$$

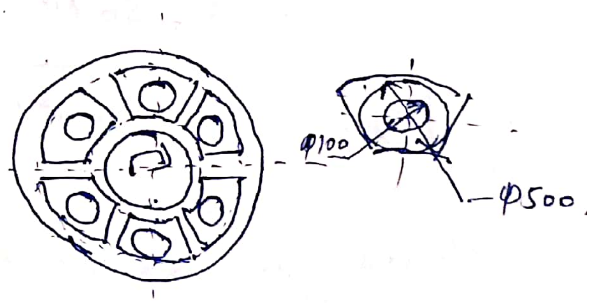
$$= 2[19.75 + 14.82]$$

$$\boxed{Q_{total} = 69.14\ mm^3/sec}$$

16.4 Problem

Given:-

No. of pads = 6.



$W = 900$ kN

$h_0 = 0.05$ mm

$\rho = 0.9$ g/cc.

~~SAE 30~~

$$Z_K = \left[0.22(300) - \frac{180}{300}\right]$$

$Z_K = 65.44$ cSt

$Z = \rho \cdot Z_K$

$Z = (0.9)(65.44) = 58.86$ cP.

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Neglect flow over corners.

~~Assume Area of Pad = 500 mm.~~ $D_o = 500\ mm$

~~Assume area ∞~~ . $D_i = 100\ mm$.

Find:- (1) $P_i = ?$

(2) $Q = ?$

Ans: Total Thrust load on six pad is 900 kN

Thrust load per pad $= \frac{900 \times 10^3}{6}$.

$$\boxed{W = 150 \times 10^3\ N.}$$

~~$W = P_i \left(\frac{\pi}{4}\right)$~~

$$P_i = \frac{2W}{\pi (R_o^2 - R_i^2)\log_e\left(\frac{R_o}{R_i}\right)}$$

$$P_i = \frac{2 \times 150 \times 10^3}{\pi [250^2 - 50^2]\log_e\left[\frac{250}{50}\right]}$$

$$\boxed{P_i = 2.56\ N/mm^2}$$

$$Q = \frac{\pi P_i h_o^3}{6 \mu \log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{\pi (2.56)(0.05)}{6(58.86 \times 10^{-9})\log_e\left(\frac{250}{50}\right)}$$

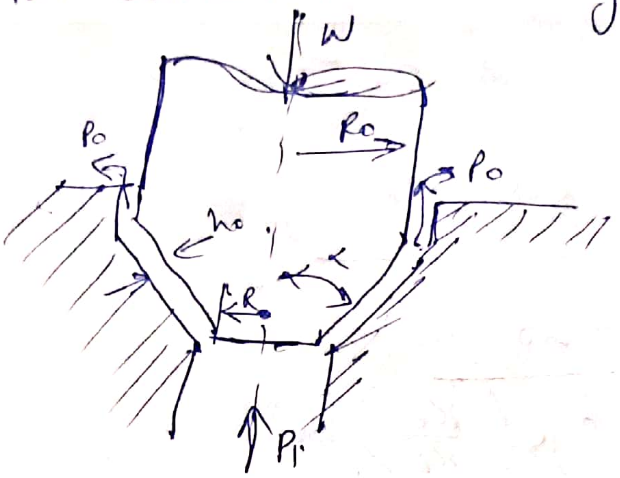
$$\boxed{Q = 1830.91\ mm^3/sec}$$

There are Six pads, Hence total flow.

$$Q_{Total} = 6 \times 1830.91$$

$$\boxed{Q_{Total} = 10985.47\ mm^3/sec}$$

Note* For Conical thrust Bearing.



$$\boxed{W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log_e\left(\frac{R_o}{R_i}\right)}\right]}$$

$$\boxed{Q = \frac{\pi P_i \cdot h_o^3 \sin\alpha}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}}$$

16.6 Problem:- Hydrostatic step Bearing of vertical turbo generator

$W = 450$ KN

$D_o = 400$ mm

$D_i = 250$ mm

$N = 750$ rpm

$Z = 30$ cP

sketch (i) Film thickness vs Energy losses

Optimum film thickness for minimum power loss.

Ans:-

$$P_i = \frac{2W \log_e\left(\frac{R_o}{R_i}\right)}{\pi(R_o^2 - R_i^2)}$$

$$P_i = \frac{2 \times 450 \times 10^3 \times \log_e\left[\frac{200}{125}\right]}{\pi(200^2 - 125^2)}$$

$$\boxed{P_i = 5.52 \ N/mm^2}$$

10

$$Q = \frac{\pi P_i h_o^3}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}$$

$$Q = \frac{\pi(5.52)(h_o)^3}{6(30 \times 10^{-9})\log_e\left(\frac{200}{125}\right)}$$

$$\boxed{Q = (205 \times 10^6) h_o^3 \ mm^3/sec}$$

Pump power

$$P_p = Q(P_i - P_o) \times 10^{-6}$$

$$= \left[(205 \times 10^6) h_o^3\right](5.52 - 0) \times 10^{-6}$$

$$\boxed{P_p = 1131.6 \cdot h_o^3}$$

Friction Power

$$P_f = \left(\frac{1}{58.05 \times 10^6}\right) \frac{\mu N^2 (R_o^4 - R_i^4)}{h_o}$$

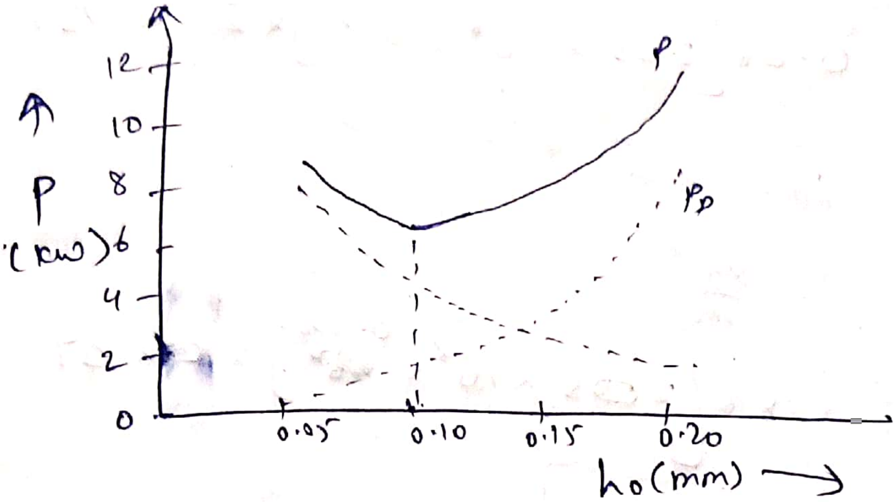
$$P_f = \left(\frac{1}{58.05 \times 10^6}\right)\left[\frac{(30 \times 10^{-9})(750)^2\left[(200)^4 - (125)^4\right]}{h_o}\right]$$

$$\boxed{P_f = \frac{0.394}{h_o}}$$

Total Power, $P = P_p + P_f$

$$\boxed{P = \cancel{1136} \ 1131.6 \ h_o^3 + \left(\frac{0.394}{h_o}\right)}$$

h_0 (mm)	P_p	P_f	P
0.05	0.14	7.8	8.02
0.10	1.13	3.94	5.07
0.1038	1.27	3.80	5.07
0.15	3.82	2.63	6.45
0.20	9.05	1.97	11.02



Differentiating P w.r.t h_o,

$$\frac{dP}{dh_o} = 3\times(1131.6)\cdot h_o^2 + \left(-\frac{0.394}{h_o^2}\right) = 0$$

$$3h_o^2(1131.6) = \frac{0.394}{h_o^2}$$

$$\boxed{h_o = 0.1038 \text{ mm}}$$

1.6.7 Problem

Given:-

$\rho = 0.86$

$C_p = 2$ kJ/kg °c

$\Delta T = ?$, $h_o = 0.1038$ mm

Assume total P lost is converted into friction heat

$$P = 1131.6 h_o^3 + \left(\frac{0.394}{h_o}\right)$$

$$= 1131.6(0.1038)^3 + \frac{(0.394)}{(0.1038)}$$

$$\boxed{P = 5.06 \text{ kW}}$$

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total Power loss = total heat Generated

$P = H = 5.06 \text{ kw} = 5.06 \ \frac{kJ}{\cancel{kg}\, sec}$

$H = m \ c_p \ \Delta T.$

$H = (\rho \cdot Q) \cdot c_p \cdot \Delta T$

$5.06 = 0.86 [205 \times 10^{6} \times 40^{3}] \times 2 \times \Delta T$

$\frac{kJ}{\cancel{kg}\, sec} \qquad \frac{\cancel{kg}}{\cancel{mm^3}} \times \frac{mm^3}{sec} \times \frac{kJ}{\cancel{kg}\, ^\circ C} \times {}^\circ C.$

$\frac{kJ}{sec} = \frac{kJ}{sec}.$

$5.06 = 0.86 [205 \times 10^{6} \times \underbrace{\quad}_{0.1038}] \times 2 \times \Delta T$

$\boxed{\Delta T = 12.84^\circ C}$

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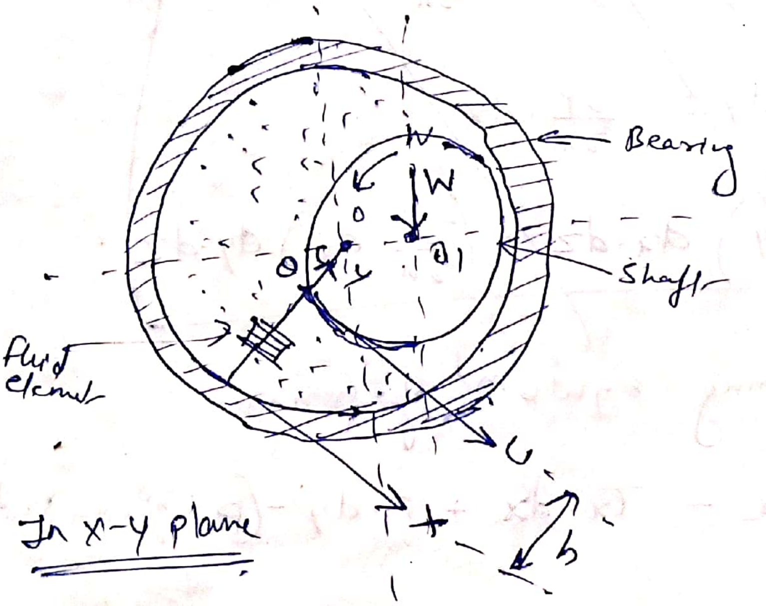
* Reynold's Equation

Theory of hydrodynamic lubrication is based on a differential equation derived by Osborne Reynold. The assumptions in this equation are.

1) Lubricant obeys Newton's law of viscosity
2) Lubricant is incompressible
3) Inertia forces in oil film are negligible.
4) Viscosity of oil is constant.
5) Effect of curvature of the film w.r.t film thickness is neglected.
6) Shaft and bearing are rigid.
7) Continous supply of lubricant is provided.



In X-Y plane

Let element of dimension dx, dy, and dz.

X → The axis in direction of motion.

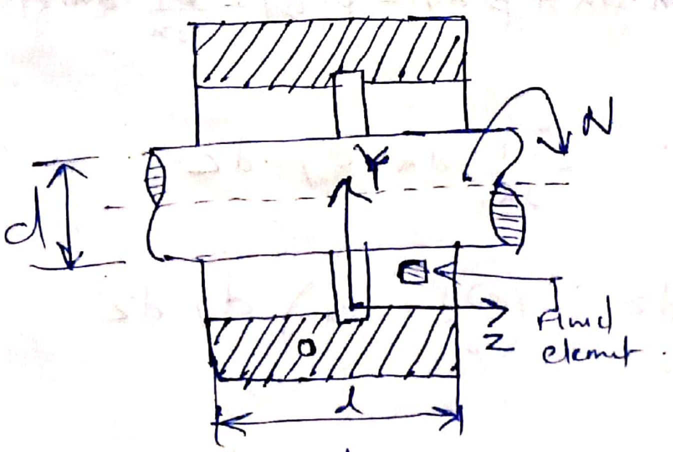
Y → Axis in radial plane.

Z → Axis parallel to axis of journal.

U, V, W velocity in X, Y, Z.

τ_x, τ_z shear stress in X & Z. direction.

p → fluid film pressure.

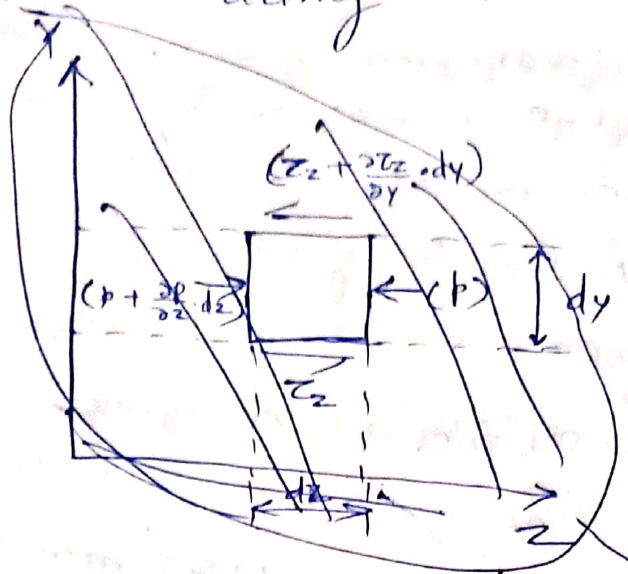


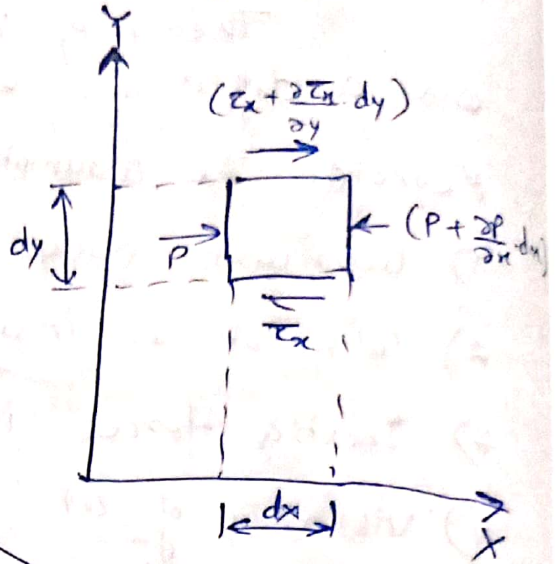
In Y-Z plane

So, forces acting on elemental ~~area~~ $\oint$, in x-dirn.

$-\tau_x \cdot dx$

$+ \tau_x dx - \partial \tau_x$





So, considering equlbm.

$$\cancel{\tau_x} + \frac{\partial \tau_x}{\partial y} \cdot dy + (P) - \cancel{\tau_x} - \left(p + \frac{\partial p}{\partial x} \cdot dx\right) = 0$$

$$\frac{\partial \tau_x}{\partial y} \cdot dy + \cancel{(p)} - \cancel{p} - \frac{\partial P}{\partial x} \cdot dx = 0$$

$$\frac{\partial \tau_x}{\partial y} \cdot dy = \frac{\partial p}{\partial x} \cdot dx$$

$$\left(\frac{\partial \tau_x}{\partial y} \cdot dy\right) dx \cdot dz = \left(\frac{\partial p}{\partial x} \cdot dx\right) dy \cdot dz$$

So considering equlbm of forces.

$$\left(\tau_x + \frac{\partial \tau_x}{\partial y} \cdot dy\right) \cdot dx - \tau_x \cdot dx + p \cdot dy - \left(p + \frac{\partial p}{\partial x} \cdot dx\right) \cdot dy = 0$$

$$\cancel{\tau_x \cdot dx} + \frac{\partial \tau_x}{\partial y} \cdot dy \cdot dx - \cancel{\tau_x \cdot dx} + \cancel{p \cdot dy} - \cancel{p \cdot dy} - \frac{\partial p}{\partial x} \cdot dx \cdot dy = 0$$

$$\frac{\partial \tau_x}{\partial y} \cdot dy \cdot dx \cdot dz = \frac{\partial P}{\partial x} \cdot dx \cdot dy \cdot dz$$

$$\left(\frac{\partial \tau_x}{\partial y} \cdot dy\right) dx \cdot dz = \left(\frac{\partial p}{\partial x} \cdot dx\right) dy \cdot dz$$

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$\because \; dx \cdot dy \cdot dz \neq 0$

Therefore.

$$\left(\frac{\partial \tau_x}{\partial y} \cdot dy\right) \cdot dx \cdot dz = \left(\frac{\partial P}{\partial x} \cdot dx\right)(dy \cdot dz)$$

$$\frac{\partial \tau_x}{\partial y} = \frac{\partial P}{\partial x}. \quad \text{— (1)}$$

By newton law, of viscosity

$$\tau_x = \mu \cdot \frac{\partial u}{\partial y} \quad \text{— (2)}$$

Substituting (2) in (1)

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \cdot \frac{\partial P}{\partial x}$$

Integrating twice.

$$\boxed{u = \frac{1}{\mu} \cdot \frac{\partial P}{\partial x} \cdot \frac{y^2}{2} + C_1 y + C_2.}$$

To find constants C_1 and C_2, ~~the~~ boundary

Conditions are, when $y=0$, $u=0$

and when $y=h$, $u=h$.

So putting first Boundary condition.

$$0 = \frac{1}{\mu} \cdot \frac{\partial P}{\partial x} \cdot \frac{(0)}{2} + C_1(0) + C_2.$$

$$\boxed{C_2 = 0}$$

Putting second Boundary condition.

$$U = \frac{1}{\mu} \frac{\partial P}{\partial x} \cdot \frac{h^2}{2} + C_1(h) + 0.$$

$$U = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{h^2}{2} + G(h)$$

$$\boxed{G = \frac{U}{h} - \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{h}{2}}$$

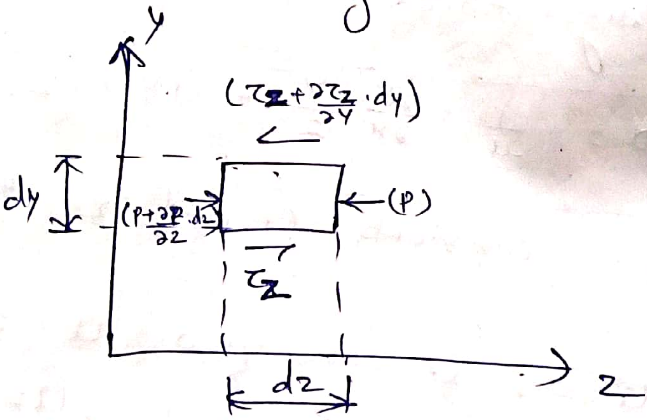
So eqution becomes

$$\cancel{u = \frac{Uy}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial}}$$

$$u = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\cdot\frac{y^2}{2} + \frac{Uy}{h} - \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\cdot\frac{h}{2}y$$

$$\boxed{u = \frac{Uy}{h} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - hy)}$$

Similarly, Forces acting on element z direction



∴ ~~Equation~~, equlbm of Forces will be.

$$-\left(\tau_z + \frac{\partial \tau_z}{\partial y}\cdot dy\right)dz + \tau_z\cdot dz + \left(p + \frac{\partial p}{\partial z}\cdot dz\right)\cdot dy - p\cdot dy = 0$$

$$-\cancel{\tau_z\, dz} - \frac{\partial \tau_z}{\partial y}\cdot dy\cdot dz + \cancel{\tau_z\, dz} + \cancel{p\cdot dy} + \frac{\partial p}{\partial z}\cdot dz\cdot dy - \cancel{p\, dy} = 0$$

$$\frac{\partial \tau_z}{\partial y}\cdot dy\cdot dz = \frac{\partial p}{\partial z}\cdot dz\cdot dy$$

$$\boxed{\left(\frac{\partial \tau_z}{\partial y}\cdot dy\right) dz\cdot dx = \left(\frac{\partial p}{\partial z}\cdot dz\right) dy\cdot dz}$$

$\therefore\ dx \cdot dy \cdot dz \neq 0.$

So,

$$\left(\frac{\partial \tau_z}{\partial y}\right) = \left(\frac{\partial p}{\partial z}\right)$$

By Newton's law,

$$\tau_z = \mu \cdot \frac{\partial w}{\partial y}$$

So,

$$\mu \cdot \frac{\partial^2 w}{\partial y^2} = \frac{\partial p}{\partial z}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu} \cdot \frac{\partial p}{\partial z}$$

Integrating twice

$$\boxed{w = \frac{1}{\mu} \cdot \frac{\partial p}{\partial z} \cdot \frac{y^2}{2} + C_3 y + C_4}$$

To find constants

B.C is,

at $y = 0$, $w = 0$

at $y = h$ $w = 0.$

First B.C.

$$0 = \frac{1}{\mu} \cdot \frac{\partial p}{\partial z}(0) + (0) + C_4$$

$$\boxed{C_4 = 0}$$

2nd B.C

$$0 = \frac{1}{\mu}\left(\frac{\partial p}{\partial z}\right)\left(\frac{h^2}{2}\right) + C_3(h) + 0.$$

$$\boxed{C_3 = -\frac{1}{\mu}\left(\frac{\partial p}{\partial z}\right) \cdot \frac{h}{2}.}$$

So equa. becomes.

$$w = \frac{1}{\mu} \cdot \frac{\partial p}{\partial z} \cdot \frac{y^2}{2} - \left(\frac{1}{\mu} \cdot \frac{\partial p}{\partial z} \cdot \frac{h}{2}\right) y$$

$$\boxed{w = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial z}\left(y^2 - hy\right)}$$

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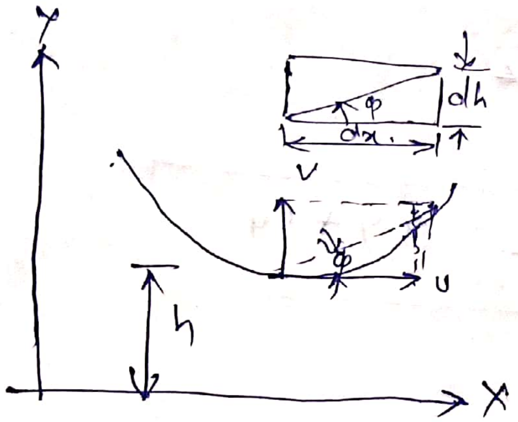
The general continuity equ^n for incompressible flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}.$$

$$\int_0^h \frac{\partial v}{\partial y} \cdot dy = -\int_0^h \frac{\partial u}{\partial x} \cdot dy - \int_0^h \frac{\partial w}{\partial z} \cdot dy$$

$$\int_0^h \frac{\partial v}{\partial y} \cdot dy = \{(v) \text{ at } (y=h)\} - \{(v) \text{ at } (y=0)\} \quad \text{— (3)}$$



The fig shows the fluid film in x-y plane.

when Y=0 → It indicates stationary bearing

Y=h → It indicates journal surface and velocity in Y-direction (V).

$$\tan\phi = \frac{V}{U} = \frac{dh}{dx}$$

$$V = U \cdot \frac{dh}{dx}.$$

Substituting equ^n (3)

$$\int_0^h \frac{\partial v}{\partial y} \cdot dy = \left[U \cdot \frac{dh}{dx} - 0\right]$$

$$\int_0^h \frac{\partial v}{\partial y} \cdot dy = U \cdot \frac{dh}{dx}$$

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$$\therefore \int_0^h \frac{\partial v}{\partial y} dy = -\int_0^h \frac{\partial u}{\partial x} \cdot dy - \int_0^h \frac{\partial w}{\partial z} \cdot dy$$

So,

$$\int_0^h \frac{\partial u}{\partial x} \cdot dy + \int_0^h \frac{\partial w}{\partial z} \cdot dy = -U \cdot \frac{dh}{dx} \longrightarrow (4)$$

we will apply Leibnitz's theorem (for interchanging the sign of integration and differentiation of the first term of the above equation, because upper limit is a function of x).

$$\frac{d}{dx} \int_{h_1(x)}^{h_2(x)} U(x,y) \cdot dy = \int_{h_1(x)}^{h_2(x)} \frac{\partial}{\partial x} u \cdot (x,y) \cdot dy$$

$$+ \left(U[h_2(x), x] \left(\frac{dh_2(x)}{dx} \right) \right)$$

$$- \left(U[h_1(x), x] \left(\frac{dh_1(x)}{dx} \right) \right)$$

Substituting the values

$h_1(x) = 0$, $h_2(x) = h$, $U(x,y) = u$

$u[h_1(x), x] = u$ at $[h_1(x), x] = 0$

$u[h_2(x), x] = u$ at $[h_2(x), x] = U$

we get,

$$\frac{d}{dx} \int_0^h u \cdot dy = \int_0^h \frac{\partial u}{\partial x} \cdot dy + u \frac{dh}{dx}.$$

Therefore first term of equation (4) becomes

$$\int_0^h \frac{\partial u}{\partial x} \cdot dy = \frac{\partial}{\partial x} \int_0^h u \cdot dy - U \cdot \frac{dh}{dx}$$

In second term of equation (4)

$$\int_0^h \frac{\partial w}{\partial z} \cdot dy = \frac{\partial}{\partial z} \int_0^h w \cdot dy$$

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Substituting both terms back in equtn (4)

$$\frac{\partial}{\partial x}\int_0^h u.dy - U\frac{dh}{dx} + \frac{\partial}{\partial z}\int_0^h w.dy = -U.\frac{dh}{dx}$$

$$\frac{\partial}{\partial x}\int_0^h u.dy + \frac{\partial}{\partial z}\int_0^h w.dy = 0 \quad \text{—(5)}$$

Substituting the value of u in equtn (5)

$$\frac{\partial}{\partial x}\int_0^h u.dy = \frac{\partial}{\partial x}\int_0^h \left[\frac{Uy}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy)\right]dy$$

$$= \frac{\partial}{\partial x}\left[\frac{Uy^2}{2h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)\right]_0^h$$

$$= \frac{\partial}{\partial x}\left[\frac{Uh}{2} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(-\frac{h^3}{6}\right)\right]$$

$$= \frac{U}{2}\frac{\partial h}{\partial x} - \frac{1}{12\mu}\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] \quad \text{—(6)}$$

Similarly,

$$\frac{\partial}{\partial z}\int_0^h w.dy = \frac{\partial}{\partial z}\int_0^h \left[\frac{1}{2\mu}\frac{\partial p}{\partial z}(y^2 - hy)\right]dy$$

$$= \frac{\partial}{\partial z}\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\left[\frac{y^3}{3} - \frac{hy^2}{2}\right]\right]_0^h$$

$$= \frac{\partial}{\partial z}\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\left[-\frac{h^3}{6}\right]\right]$$

$$= -\frac{1}{12\mu}\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] \quad \text{—(7)}$$

Substituting (6) & (7) in (5)

$$\frac{U}{2}\frac{\partial h}{\partial x} - \frac{1}{12\mu}\frac{\partial}{\partial x}\left[h^3.\frac{\partial p}{\partial x}\right] - \frac{1}{12\mu}\frac{\partial}{\partial z}\left[h^3.\frac{\partial p}{\partial z}\right] = 0$$

or,

$$\boxed{\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] = 6\mu U\left(\frac{\partial h}{\partial x}\right)}$$

The above equtn is known as Reynold's equation. There is no exact analytical solution for this equation for bearing with finite length. exact solution can be obtained if bearing is assumed to be either infinitely long or very short. These two solutions are called Sommerfeld Solutions.

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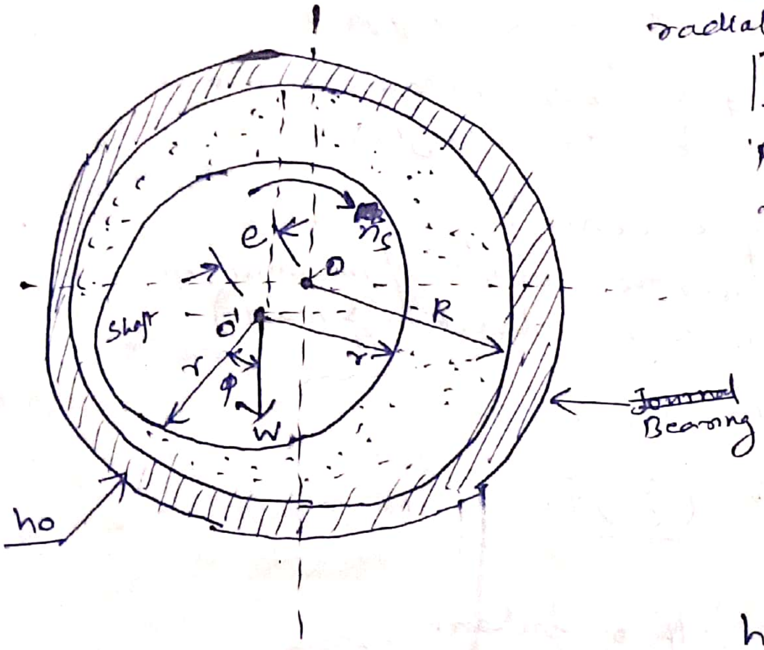
* Raimondi and Boyd Method:-

There is no exact solution to Reynold's equation for a journal bearing having a finite length.

A.A Raimondi and John Boyd of westinghouse Research laboratory solved this equation on computer using iteration technique.

The performance of bearing is expressed in terms of dimensionless parameters.

This is given in table values of these parameters for a full journal bearing with side flow.



$OO' = e$

radial clearance,

$$\boxed{C = R - r}$$

R → Radius of bearing

r → — — shaft.

eccentricity ratio,

$$\boxed{\epsilon = \frac{e}{c}}$$

So from the fig.

$$\boxed{R = e + r + h_o}$$

h_o = minimum film thickness

$$C = R - r$$

$$= e + r + h_o - r$$

$$C = e + h_o$$

$$C = (\epsilon C + h_o) \quad \text{—— (1)}$$

$$C - \epsilon C = h_o$$

$$\boxed{C(1-\epsilon) = h_o}$$

~~diving~~ dividing equn (1) with C

$$1 = \epsilon + \frac{h_o}{c}$$

$$\boxed{\epsilon = 1 - \left(\frac{h_o}{C}\right)}$$

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The quantity $\left(\frac{h_o}{c}\right)$ is called the minimum film thickness variable.

The sommerfield number is given by,

$$\boxed{S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{P}}$$

S = Sommerfield number (dimensionless)
μ = viscosity of the lubricant ($N\text{-}s/mm^2$)
n_s = journal speed (rev./s)
p = unit bearing pressure (N/mm^2)

The sommerfield number contains all variables, which are controlled by the designer.

ϕ = Angle of eccentricity
(It locates the position of minimum film thickness w.r.t direction of load)

Coefficient of friction variable,

$$\boxed{CFV = \left(\frac{r}{c}\right) f}$$

where,

f = Coeff. of friction.

frictional torque,

$$\boxed{(M_t)_f = f \cdot Wr} \quad N\text{-}mm.$$

Frictional power,

$$= (2\pi n_s)(fW \cdot r) \quad N\text{-}mm/sec$$
$$= (2\pi n_s)(fW \cdot r) \times 10^{-3} \, W.$$
$$= (2\pi n_s)(fW \cdot r) \times 10^{-6} \, kW.$$

Therefore

Power, $$\boxed{P_f = \frac{2\pi n_s (fW) \cdot r}{10^6}}$$

Flow variable,

$$\boxed{FV = \frac{Q}{rc n_s l}}$$

where l = length of bearing (mm)
Q = flow of lubricant mm^3/sec

Q (Total flow of lubricant oil) → one part of Q is circulated around the journal
→ another part is flows out as side leakage (Q_s)

The Q_s can be calculated from the parameter ($\frac{Q_s}{Q}$) given in the table.

The P_{max} is calculated from ratio $\left(\frac{p}{p_{max}}\right)$

Temperature Rise :-

Heat is generated in the bearing due to viscosity of lubricating oil.

Frictional work → heat → increases temp. of lubricant.

Assuming total heat generated in the bearing is carried away by total oil flow in the bearing.

$$P_f = (2\pi n_s)(f W \cdot r) \times 10^{-6}.$$

∴ Heat generated (H_g)

$$H_g = P_f = (2\pi n_s)(f W \cdot r) \times 10^{-6} \quad kW,$$

$$\therefore f = \left(\frac{c}{r}\right)(CFV) \quad \text{and} \quad W = 2pLr$$

$$H_g = 2\pi n_s \left(\frac{c}{r}\right)(CFV)(2pLr) \cdot r \times 10^{-6}$$

$$\boxed{H_g = (4\pi)(10^{-6})\, r\, c\, n_s\, l\, p\, (CFV)}$$

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Heat carried away by oil flow (Hc) is given by,

$$H_c = m C_p \Delta t$$

$$= \cancel{(\rho \cdot Q)\, C_p\, \Delta t}$$

mass of lubricating oil,

$$m = \rho Q (10^{-6}) \text{ kg/sec}$$

$$\therefore Q = r c n_s \cdot l (FV)$$

$$m = \rho \cdot r c \cdot n_s \cdot l (FV)(10^{-6})$$

$$\boxed{H_c = C_p \Delta t (\rho)(r n_s c l)(FV)(10^{-6})}$$

Equating both

i.e $H_c = H_g$

$$C_p \cdot \Delta t (\rho)(\cancel{r n_s} \cdot \cancel{c l})(FV)\cancel{(10^{-6})} = (4\pi)\cancel{(10^{-6})}\cancel{r \cdot c \cdot n_s}\,(\rho)(CFV)$$

$$\boxed{\Delta t = \left(\frac{4\pi p}{\rho C_p}\right)\frac{(CFV)}{(FV)}}$$

for most lubricating oils,

$\rho = 0.86$ & $C_p = 1.76$ kJ/kg°C.

$$\boxed{\Delta t = \frac{8.3 p (CFV)}{(FV)}}$$

Avg temp. of lubricating oils,

$$\boxed{T_{av} = T_i + \frac{\Delta t}{2}}$$

T_i = Input oil temp.

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* Bearing Design - Selection of Parameters

In preliminary ~~stag~~ stages of journal bearing design it is required to select suitable values for following parameters :-

1. Length - to - diameter ratio.
2. Unit bearing pressure
3. Start - up load.
4. radial clearance.
5. minimum oil film thickness.
6. maximum oil film temperature.

(1) Length to Diameter Ratio:-

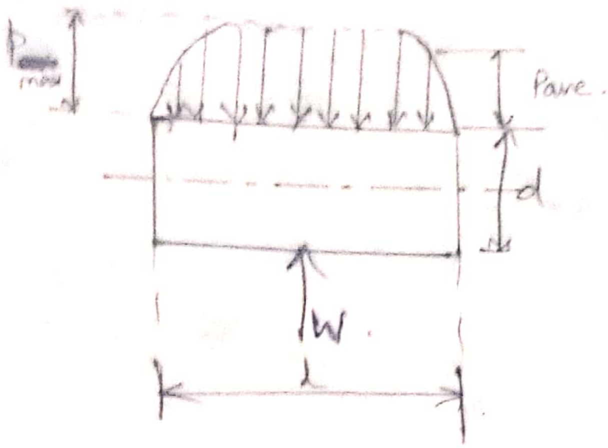
In hydrodynamic bearing,

- Diameter of shaft is determined by strength or rigidity considerations, and not on bearing capacity
- The shaft diameter is determined by criteria such as permissible stress, permissible lateral deflection or ~~permi~~ permissible twist.

Therefore the designer can only vary the bearing length.

- The length to diameter (l/d) affects the bearing performance.
- As ratio increases the resulting film increases

Long Bearing



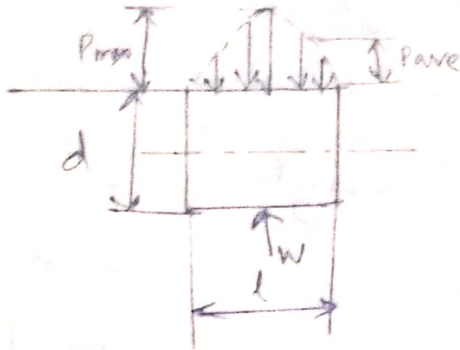
$\frac{l}{d} = 2$

1) It is has more load carrying capacity compared to short Bearing.

2) long bearings are more suspectible to metal to metal contact at the two edges, when shaft is deflected under load

3) It is difficult to get sufficient oil flow through the passage.

Short Bearing



$\frac{l}{d} = 1$

1) A ~~sode~~ short bearing has greater side flow, which improve heat dissipation.

2) They are less

3) Easy to get sufficient oil flow.

Therefore design trend is to use $\left(\frac{l}{d}\right) \leq 1$.

$\left(\frac{l}{d}\right)$ can be taken 1, when

- shaft and bearing is precesily alligned
- shaft deflection is in limit
- Cooling of lubricant and bearing doesn't present a serious problem.

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In practice,

$\left(\frac{l}{d}\right)$ varies from 0.5 to 2.0.

but in majority applications it is taken as 1.

Terminology :

1) if $\left(\frac{l}{d}\right) > 1$ $\longrightarrow$ long bearing

2) if $\left(\frac{l}{d}\right) < 1$ $\longrightarrow$ short Bearing.

3) if $\left(\frac{l}{d}\right) = 1$ $\longrightarrow$ Square Bearing.

② Unit Bearing Pressure

$$\text{Unit Bearing Pressure} = \frac{\text{Load}}{(\text{Projected area of Bearing in running condition.})}$$

It depends upon following factors :

a) Bearing material

b) operating temperature

c) Nature and frequency of load.

d) Service conditions.

	Application	Unit Bearing Pressure. (N/mm^2)
i)	Diesel Engine	
	Main bearing	5-10
	crank pin	7-14
	Gudgeon Pin	13-14
ii)	Automotive Engine	
	Main Bearing	3-4
	Crank pin	10-14

iii)	Air compressors	
	Main bearing	1 - 1.5
	crank pin	1.5 - 3.0
iv)	centrifugal Pumps Main Bearing	0.5 - 0.7
v)	Electric Motors Main Bearing	0.7 - 1.5
vi)	Transmission shafts	
	Light duty	0.15
	Heavy duty	1.00
vii)	Machine tools Main Bearing	2

③ **start-up load** :-

The unit bearing pressure for starting conditions should not exceed $2 N/mm^2$.

The startup is static load, when shaft is stationary.

$$\text{Startup load} = \left(\text{Dead wt of Shaft}\right) + \left(\text{wt. of attachments}\right)$$

The startup load can be used to determine the minimum length of the bearing on the basis of starting conditions.

(4) Radial clearance :-

Radial clearance should be small to provide the necessary velocity gradient.

To provide small clearance it requires

- costly finishing operations
- rigid mountings of the bearing assembly
- Clean lubricating oil ~~witho~~ without any foreign particles.

This requires initial and maintenance costs.

$$\boxed{c = (0.001)r \text{ mm}}$$

material	Radial clearance
Babbits	$(0.001)r$ to $(0.00167)r$
Copper lead	$(0.001)r$ to $(0.01)r$
Alluminium Alloy	$(0.002)r$ to $(0.0025)r$.

(5) Minimum Oil film thickness :-

— The surface finish of the journal and the bearing is governed by the value of the minimum oil film thickness selected by the designer & vice versa.

— There is lower limit for minimum oil thickness below which metal to metal contact occurs and hydrodynamic film breaks

The lower limit is,

$$\boxed{h_o = (0.0002)r}$$

(8) maximum oil film temperature

~~The lubricating~~

when, operating temp > 120°c, lubricating oil oxidise

(1) at operating temp > 125°c and Bearing pressure = 7 N/mm² } Babit tends to soften.

(2) at operating temp > 190°c } Bearing pressure = 1.4 N/mm²

Therefore operating temperature should be kept within these limits.

In general limiting temperature is 90°c for Bearings made of Babbitts.

Bearing can be designed for two different conditions:-

(1) Bearing for max^m load carrying capacity

(2) Bearing for minimum frictional loss

l/d	$\left(\frac{h_o}{c}\right)$ for maximum load	$\left(\frac{h_o}{c}\right)$ for minimum friction
∞	0.66	0.60
1	0.53	0.30
0.5	0.43	0.12
0.25	0.27	0.03.

Ques
169
Problem

Given:-

$W = 3.2 kW$

$n_s = 1490$ rpm

$d = 50$ mm

$l = 50$ mm

$c = 0.05$ mm

$z = 25$ cP

Find:-

1) $f = ?$

2) $P_f = ?$

3) $h_o = ?$

4) $Q = ?$

5) $\Delta t = ?$

Ans: $p = \frac{W}{ld}$

$= \frac{3.2 \times 10^3}{(50)(50)}$

$$\boxed{p = 1.28 \ N/mm^2}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$

$$= \left(\frac{25}{0.05}\right)^2 \left(\frac{25}{10^9}\right)\left(\frac{1490}{60}\right)\left(\frac{1}{1.28}\right)$$

$$\boxed{S = 0.121}$$

$$\frac{l}{d} = \left(\frac{50}{50}\right) = 1$$

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from table

$\left(\frac{r}{c}\right)f = 3.22$, $\frac{h_o}{c} = 0.4$, $\left(\frac{Q}{r c n_s l}\right) = 4.33$

Coefficient of friction. (f)

$f = 3.22\left(\frac{c}{r}\right)$

$= 3.22\left(\frac{0.05}{25}\right)$

$\boxed{f = 0.00644}$

Power lost in friction

$P_f = \frac{2\pi n_s (f W r)}{10^6}$

$= \frac{2\pi(1490)(0.00644)(3.2\times10^3)(25)}{60 \times 10^6}$

$\boxed{P_f = 0.08 \text{ W}}$

Minimum oil film thickness,

$h_o = 0.4\, c$

$= 0.4(0.05)$

$\boxed{h_o = 0.02 \text{ mm}}$

Flow requirement

$Q = 4.33\, r c n_s l$

$= (4.33)(25)(0.05)\left(\frac{1490}{60}\right)(50)$

$\boxed{Q = 6720.5 \text{ mm}^3/\text{sec}}$

$Q = 6720.5 \times 10^{-3}$ cm³/s

$= 6720.5 \times 10^{-3} \times 10^{-3}$ ~~litre~~ litre/sec $(\because 1$ litre $= 10^3$ cc$)$

$= 6720.5 \times 10^{-6}$ litre/sec

$Q = 6720.5 \times 10^{-6} \times 60$ litre/min

$$\boxed{Q = 0.403 \text{ litre/min}}$$

Temp. rise

$$\Delta t = \frac{8.3\,p\,(CFV)}{FV}$$

$$= \frac{8.3\,(1.28)(3.22)}{(4.33)}$$

$$\boxed{\Delta t = 7.9\,°C}$$

16.10 Given:-

$W = 1200$ N

$n_s = 1440$ rpm

$d = 50$ mm

static load $= 350$ N

$h_o = 5$ (Sum of surface roughness)

CLA → centerline average

2 and 1 means $(2+1) = 3$, m

To find

1) $l = ?$

2) $c = ?$

3) $h_o = ?$

4) $\mu = ?$

5) $Q = ?$

length of Bearing

During starting condition,

$$p = \frac{W}{ld}$$

$$(2) = \frac{350}{l \cdot (50)}$$

$$\boxed{l = 3.5mm}$$

[∴ As we know that unit bearing pressure for starting condition should not increase $2 N/mm^2$]

During Running Condition:-

$$p = \frac{W}{ld}$$

$$\cancel{\frac{1200}{(3.5)(50)}}$$

from table the permissible bearing pressure is 0.7 to $1.5 N/mm^2$ so we assum $1 N/mm^2$.

$$(1) = \frac{1200}{(l)(50)}$$

$$\boxed{l = 24mm}$$

So from both condition this length is more hence we will take $\boxed{l = 24mm}$ only.

so,

$$\frac{l}{d} = \frac{24}{50} = 0.48$$

we will assume standard value for $\left(\frac{l}{d}\right) = 0.5$

$$l = 0.5(d)$$

$$= 0.5(50)$$

$$\boxed{l = 25mm}$$

So we select length of bearing as

$$\boxed{l = 25mm}$$

2) Radial clearance (c)

$c = 0.001(r)$

$= 0.001(25)$

$= \cancel{0.0025}$

$\boxed{c = 0.025 \text{ mm}}$

3) minimum oil film thickness (ho)

ho = 5 × (sum of roughness values)

$= 5 \times (2+1)$

$= 15$ microns.

$= 15 \times 10^{-3}$ mm

$\boxed{h_o = 0.015 \text{ mm}}$

4) viscosity of lubricant

$\therefore \left(\frac{l}{d}\right) = \frac{1}{2}$. and $\frac{h_o}{c} = \frac{0.015}{0.025} = 0.6$

So, from table.

$S = 0.779$ and $\frac{Q}{rcn_s l} = 4.29$.

$n_s = \frac{1440}{60} = 24$ rps.

$p = \frac{W}{ld} = \frac{1200}{(25)(50)} = 0.96 \text{ N/mm}^2$

$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu n_s}{p}\right)$

$0.779 = \left(\frac{25}{0.025}\right)^2 \left(\frac{\mu \cdot (24)}{0.96}\right)$

$\mu = (31.16 \times 10^{-9}) \frac{N\text{-sec}}{mm^2}$

$\boxed{\mu = 31.16 \text{ cP}}$

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5) Selection of lubricant :-

From viscosity vs temp relationship
for $t = 65^\circ$ at $\mu = 31.16$ cP.

Values of SAE 30 and SAE-40 are suitable
i.e. 30 cP and 38 cP respectively.

Hence we select

SAE 40. oil

6) Flow of lubricant-

$$Q = 4.29\, r c n_s l$$
$$= (4.29)(25)(0.025)(24)(25)$$
$$\boxed{Q = 1608.75\ mm^3/s}$$

16.11 Problem

Given:-

$d = 75mm$

$W = 10 kN$

$n_s = 1440$ rpm

$h_o = 22.5$ microns $= 22.5 \times 10^{-3}$ mm.

$T_i = 40^\circ C$

Bearing mtrl. = Babbit

$l = ?$

Select oil.

$$p = \frac{W}{ld}$$

$$2 = \frac{10 \times 10^3}{(l)(75)}$$

(For m/c tool application $p = 2 N/mm^2$)

$$\boxed{l = 66.67mm}$$

$\therefore \left(\frac{l}{d}\right) = \frac{66.67}{75} = 0.88$

we select standard $\left(\frac{l}{d}\right)$ ratio.

i.e $\boxed{\frac{l}{d} = 1}$

$l = (1)(d)$

$l = (1)(75)$

$\boxed{l = 75 mm}$

Selection of lubricant

$p = \frac{W}{ld}$

$= \frac{10,000}{(75)(75)}$

$\boxed{p = 1.78 \, N/mm^2}$

$c = (0.001) r$

$= (0.001)\left(\frac{75}{2}\right)$

$\boxed{c = 0.0375 \, mm}$

$\frac{h_o}{c} = \frac{22.5 \times 10^{-3}}{0.0375} = 0.6$

$\boxed{\frac{h_o}{c} = 0.6}$

so for $\frac{h_o}{c} = 0.6$ and $\frac{l}{d} = 1$

from table, $S = 0.264$, $\left(\frac{r}{c}\right)f = 5.79$, $\frac{Q}{rcn_s l} = 3.99$

$$n_s = \frac{1440}{60} = 24 \text{ rps.}$$

$$S = \left(\frac{r}{c}\right)^2\left(\frac{\mu n_s}{p}\right)$$

$$0.264 = \left(\frac{75}{2\times 0.0375}\right)^2\left(\frac{\mu \cdot (24)}{1.78}\right)$$

$$\boxed{\mu = 19.58\times 10^{-9}\, N\text{-}s/mm^2}$$

or

$$\boxed{\mu = 19.58 \text{ cP}}$$

~~from~~ and temp difference,

$$\Delta t = \frac{8.3p\,(CFV)}{(FV)}$$

$$\Delta t = \frac{8.3\,(1.78)(5.79)}{(3.99)}$$

$\therefore CFV = \left(\frac{r}{c}\right)f$

$FV = \frac{Q}{rcn_sl}$

$$\boxed{\Delta t = 21.44^\circ C}$$

$$T_{av} = T_i + \frac{\Delta t}{2}$$

$$= 40 + \frac{21.44}{2}$$

$$\boxed{T_{av} = 50.72^\circ C}$$

So from viscosity Vs temp. graph,

at $\mu = 19.58$ cP. and $T_{av} = 50.72^\circ C$

we select

<u>SAE-10 Oil</u>

16.12
Problem

Given:-

$\frac{l}{d} = 1$

$n_s = 1350$ rpm

$d = 100$mm

dia. clearance $= 100\,\mu m = 100 \times 10^{-3}$ mm $=$ ~~0.1mm~~ 0.1 mm

$c = \frac{0.1}{2} = 0.05$ mm.

$W = 9$ kN

~~ho = 0.3~~ minimum film thickness variable $\left(\frac{h_o}{c}\right) = 0.3$

$\mu = ?$

Ans:-

$$\frac{h_o}{c} = \frac{0.3}{0.1} = 0.3.$$

$$\frac{l}{d} = 1$$

from table.

for

	h_o/c	S.
	0.4	0.121
0.3 →		← ?
	0.2	0.0446

$$S = \frac{0.121 + 0.0446}{2}.$$

$$\boxed{S = 0.0828}$$

$$p = \frac{W}{ld}$$

$$= \frac{9 \times 10^3}{(100)^2}$$

$$\boxed{p = 0.9\ N/mm^2}$$

so,

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$

$$0.0828 = \left(\frac{50}{0.05}\right)^2 \frac{\mu\,(1350)}{(60 \times 0.9)}$$

$\mu = 3.312 \times 10^{-9}$ N-s/mm²

or

$\boxed{\mu = 3.312 \text{ cP}}$.

16.13 Problem

360° Hydrodynamic Bearing

$W = 10$ kN

$n_s = 1440$ rpm

$p = 1000$ kPa $= 1000 \times 10^3$ N/m²

$= 1000 \times 10^3 \times 10^{-6}$ N/mm²

$= 1000 \times 10^{-3}$ N/mm²

$= 1$ N/mm².

Clearance ratio, $\frac{r}{c} = 800$

$\mu = 30$ mPa s

$\cancel{= 30 \text{ N/mm}^2 \text{ sec}}$ $\cancel{30 \times 10^{-9} \text{ cP}}$. $= 30$ mill Pa-sec

$= 30 \times 10^{-9}$ MPa.

$= 30 \times 10^{-9}$ N-sec/mm²

$\mu = 30$ cP

1 mill Pa $= 10^{-3}$ Pa.

1 Mpa $= 10^6$ Pa

so,

1 milliPa $= (10^{-3})(10^{-6})$ M

1 mill Pa $= 10^{-9}$ MPa.

Assume $\underline{H_g = H_c}$. (100% heat transfered)

1) $\frac{l}{d} = ?$

2) $f = ?$

3) $P_f = ?$

4) $Q = ?$

5) Side leakage $= ?$

6) $\Delta t = ?$

Assume $\frac{l}{d} = 1$.

$p = \frac{W}{ld}$

Scanned with CamScanner

$$f = \frac{10\times10^3}{(l)(d)}$$

$\therefore l = d$

$$f = \frac{10\times10^3}{(l)^2}$$

$l = 100mm$

$$\boxed{l = d = 100mm}$$

$$S = \left(\frac{r}{c}\right)^2\left(\frac{\mu n_s}{p}\right)$$

$$S = (800)^2\left[\frac{(30\times10^{-9})(1440)}{(60)(1)}\right]$$

$$\boxed{S = 0.4608}$$

l/d	S	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{rcn_s l}\right)$	$\frac{Q_s}{Q}$
1.	0.631	12.8	3.59	0.280
	0.4608	?	?	
	0.264	5.79	3.99	0.497

So by interpolation

$$\frac{12.8 - 5.79}{0.631 - 0.264} = \frac{\left(\frac{r}{c}\right)f - 5.79}{(0.4608) - 0.264}$$

$$\boxed{\left(\frac{r}{c}\right)f = 9.55}$$

So,
$$f = \frac{9.55}{(r/c)} = \frac{9.55}{800}$$
$$\boxed{f = 0.0119}$$

Similarly

$$\frac{3.99 - 3.59}{0.264 - \cancel{0.4608}\ 0.631} = \frac{\left(\frac{Q}{r c n_s l}\right) - 3.59}{0.4608 - \cancel{0.4608}\ 0.631}$$

$$\boxed{\left(\frac{Q}{r c n_s l}\right) = 3.78.}$$

Similarly,

$$\frac{0.280 - 0.497}{0.631 - 0.264} = \frac{\left(\frac{Q}{Q_s}\right) - 0.497}{0.4608 - 0.264}$$

$$\boxed{\frac{Q}{Q_s} = 0.38}$$

Power lost in friction

$$P_f = \frac{2\pi n_s . (f W) r}{10^6}$$

$$= \frac{2\pi \times (1440)(0.0119 \times 10000)(50)}{60 \times 10^6}.$$

$$\boxed{P_f = 0.9\ W}$$

Total flow of oil.

$\therefore \frac{r}{c} = 800$

$c = \frac{r}{800}$

$c = \frac{50}{800}$

$$Q = 3.78\ r c n_s l$$

$$= (3.78)(80)\left(\frac{50}{800}\right)\left(\frac{1440}{60}\right)(100)$$

$$\boxed{Q = 28350\ mm^3/sec}$$

Side leakage:-

$$Q_s = 0.38\ Q$$

$$\boxed{Q_s = 10773\ mm^3/sec}$$

Temp. rise :-

$$\Delta t = \frac{8.3 p (CFV)}{FV} = \frac{8.3(1)(9.55)}{3.78} = 22.97^\circ C$$

$$\boxed{\Delta t = 22.97^\circ C}$$

16.14 Problem

Given:-

$W = 6.5\ kN$

$n_s = 1200 rpm$

$d = 60 mm$

$l = 60 mm$

$h_o = 0.009 mm$

$\mu = ?$

Class of fit: H7e7 (fine) running fit.

Hole limits (60+ 0.00) and (60+ 0.03) mm

Shaft limits (60 - 0.09) and (60 - 0.06) mm.

If manufacturing processes are centered, the average diameters

$$\text{Hole diameter} = \frac{60.00 + 60.03}{2} = 60.015 mm$$

$$\text{Shaft diameter} = \frac{59.91 + 59.94}{2} = 59.925 mm$$

$$c = \frac{1}{2}(60.015 - 59.925)$$

$$\boxed{c = 0.045\ mm}$$

$$\left(\frac{h_o}{c}\right) = \frac{0.009}{0.045} = 0.2$$

$$\left(\frac{l}{d}\right) = \frac{60}{60} = 1$$

So from table

$S = 0.0446$.

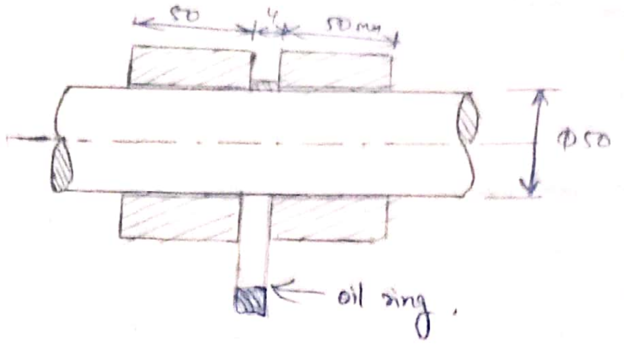
$$\mu = S\left(\frac{c}{r}\right)^2\left(\frac{P}{n_s}\right)$$

$$= (0.0446)\left(\frac{0.045}{30}\right)^2\left(\frac{1}{1200/60}\right)$$

$$\mu = 9.06 \times 10^{-9}\ N\text{-}s/mm^2$$

$$\boxed{\mu = 9.06\ cP}$$

16.15 Problem



Given:-

$W = 20\ kN$

$n_s = 1450\ rpm$

$c = 20\ micron = 20 \times 10^{-3}\ mm = 0.020\ mm$

$h_o = 5\ micron = 0.005\ mm$

① $\mu = ?$

② $Q = ?$

$$\left(\frac{h_o}{c}\right) = \frac{0.005}{0.02} = 0.25$$

$$\frac{l}{d} = \frac{50}{50} = 1.$$

Since load is divided in two Bearings. So, W=10 kN.

$$P = \frac{W}{dl}$$

$$= \frac{10 \times 10^3}{(50)(50)}$$

$$\boxed{P = 4 \ N/mm^2}$$

$\frac{l}{d}$	$\frac{h_o}{c}$	S	$\frac{Q}{rcn_s l}$
1	0.4	0.121	4.33
	0.25	?	?
	0.2	0.0446	4.62

$$\frac{0.2 - 0.4}{0.25 - 0.4} = \frac{0.0446 - 0.121}{(S) - 0.121}$$

$$\boxed{S = 0.0637}$$

$$\frac{0.2 - 0.4}{0.25 - 0.4} = \frac{4.62 - 4.33}{\left(\frac{Q}{rcn_s l}\right) - 4.33}.$$

$$\left(\frac{Q}{rcn_s l}\right) = 4.54$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{P}$$

$$0.0677 = \left(\frac{25}{0.02}\right)^2 \frac{(\mu)(1450)}{60 \times 4}$$

$$\mu = 6.74 \times 10^{-9} \ N\text{-}s/mm^2$$

$$\boxed{\mu = 6.74 \ cP.}$$

Lubricant flow:-

$$\frac{Q}{rcn_s . l} = 4.54$$

$$Q = 4.54 \times (25)(0.02)\left(\frac{1450}{60}\right)(50)$$

$$\boxed{Q = 2.74 \times 10^3 \ mm^3/sec}$$

Since there are two Bearings so total flow of oil

$$Q_{total} = 2Q$$

$$\boxed{Q_{total} = 5.48\cancel{8} \times 10^3 \ . \ mm^3/sec}$$

1 cm = 10 mm

10^{-1} cm = 1 mm

1 min = 60 sec

$\frac{1}{60}$ min = 1 sec

1 litre = 10^3 cc

10^{-3} litre = 1 cc

$$Q_{total} = 5485.83 \ mm^3/sec$$

$$= 5485.83 \times 10^{-3} \ cm^3/sec$$

$$= 5485.83 \times 10^{-3} \times 60 \ cm^3/min$$

$$= 5485.83 \times 10^{-3} \times 60 \times 10^{-3} \ litr/min$$

$$\boxed{Q = 0.321 lit/min} = 329150 \times 10^{-6} \ litr/min$$

16.16

Problem

$d = 50mm$

$n_s = 1440 rpm$

$l = 50mm$

$h_o = 5$ (Roughness of Journal and Bearing)

	M/c method	Surface Roughness (cla)
Shaft	grinding	0.8 microns
Bearing	turning/ Boring	1.6 microns

Class of fit - H8d8

$1 cP = 10^{-9} N\text{-}s/mm^2$

$Z = 18 cP.$

$\mu = 18 \times 10^{-9} N\text{-}s/mm^2$

$W = ?$

H8d8	lower limit	upper limit
For Hole :	$50 + 0.00 = 50.00$ mm	$50 + 0.039 = 50.039$ mm
For Shaft :	$50 - 0.119 = 49.8810$ mm	$50 - 0.080 = 49.92$ mm

average size :

$$\text{Hole} = \frac{50 + 50.039}{2} = 50.0195 \text{ mm}$$

$$\text{Shaft} = \frac{49.8810 + 49.92}{2} = 49.9005 \text{ mm}$$

$$c = \frac{1}{2}(50.0195 - 49.9005)$$

$$\boxed{c = 0.0595 \text{ mm}}$$

$$h_o = 5(0.8 + 1.6) \times 10^{-3}$$

$$\boxed{h_o = 0.012 \text{ mm}}$$

$$\frac{h_o}{c} = \frac{0.012}{0.0595} = 0.2017$$

$$\frac{l}{d} = \frac{50}{50} = 1.$$

From table,

$$S = 0.0446$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{P}.$$

$$0.0446 = \left(\frac{25}{0.0595}\right)^2 \frac{(18\times10^{-9})(1440)}{60\times P}$$

$$P = 1.71 \text{ N/mm}^2$$

$$P = \frac{W}{ld}$$

$$1.71 = \frac{W}{(50)(50)}$$

$$\boxed{W = 4274.98 \text{ N}}$$

16.17 Problem

Given:-

$W = 2kN$.

$d = 50mm$

$l = 50mm.$

$\mu = 20 mPa\ sec$

$\mu = 20\times10^{-9}$ N-sec/mm²

$c = ?$

$n_s = 2800 rpm.$

$h_o = 30 micron = 0.03 mm$

$f = ?$

$1m = 10^3 mm$

1 mill Pa sec $= 10^{-3}$ Pa sec
$= 10^{-3}$ N/m² sec
$= 10^{-9}$ N

$$\frac{l}{d} = \frac{\cancel{2}50}{50} = 1$$

$$p = \frac{W}{ld}$$

$$= \frac{2 \times 10^3}{(50)(50)}$$

$$\boxed{p = 0.8\ N/mm^2}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$

$$S = \left(\frac{25}{c}\right)^2 \frac{(20 \times 10^{-9})(2800)}{60 \times 0.8}.$$

$$\boxed{S = \frac{0.73 \times 10^{-3}}{c^2}.}$$

Since two things are unknown, we need to solve the problem by trial & error.

Trial 1: let assume $\left(\frac{h_0}{c}\right) = 0.9$

$$c = \frac{h_0}{0.9}$$

$$\boxed{c = 0.0333\ mm.}$$ — assumed value

From table,

$$S = 1.33$$

$$c = \sqrt{\frac{0.73 \times 10^{-3}}{S}}$$

$$\boxed{c = 0.0234\ mm.}$$ — calculated value.

Since there is difference

Trial 2 assume $\frac{h_o}{c} = 0.6$

$$c = \frac{h_o}{0.6}$$

$$= \frac{0.03}{0.6} = 0.05 \text{ mm}$$

$$\boxed{c = 0.05 \text{ mm}}$$ — assumed value

from table 16.1 $S = 0.264$

$$c = \sqrt{\frac{0.73 \times 10^{-3}}{0.264}}$$

$$\boxed{c = 0.0525 \text{ mm}}$$ — calculated value

Since both are close hence assumption is correct

$c = 0.05$ mm and $\frac{h_o}{c} = 0.6$

Coeff. of friction.

$$\left(\frac{r}{c}\right) f = 5.79$$

$$f = \frac{5.79 \times 0.05}{25}$$

$$\boxed{f = 0.01158}$$

Rolling Contact Bearings

Bearing is the mechanical element that permits relative motion between two parts such as shaft and the housing with minimum friction.

Functions of Bearings :-

1) Bearing ensures free rotation of shaft or the axle with minimum friction.

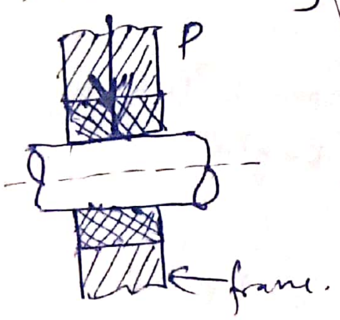
2) Bearing supports the axle or shaft.

3) Bearing takes up the forces that act on the shaft & transmit them to frame or foundation.

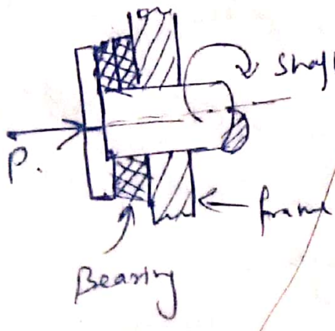
Bearings

Depending on direction of force

1) Radial Bearing



2) Thrust Bearing



Depending on type of friction

1) Sliding Contact — In this shaft slides on Bearing and hence wear takes place. In order to reduce wear film of lubricating oil is provided.

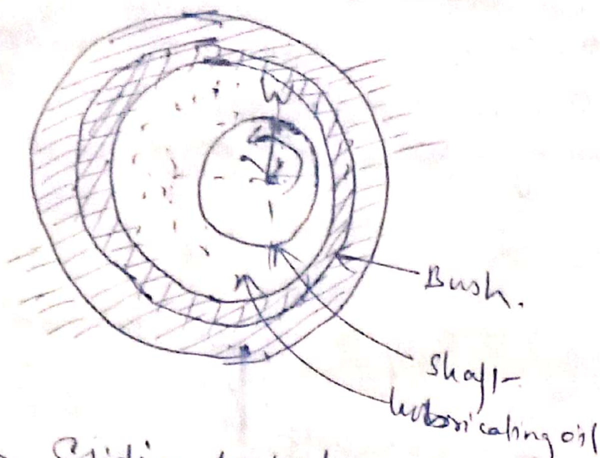
They are used in following applications

1) Crankshaft bearing in petrol & diesel Engines
2) Centrifugal pump.
3) large size electric motors.
4) Steam and gas turbine
5) Concrete mixers, rope conveyers and marine Installations.

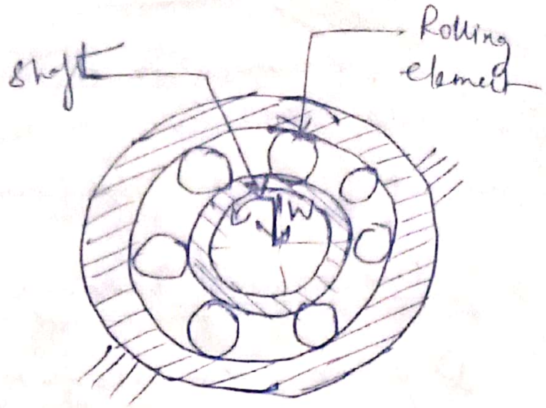
2) Rolling contact Bearing (This are also called antifriction Bearings or simply Ball Bearings.

The rolling Bearing are used in :-

(i) machine tool spindles
ii) automobile front and rear axles
iii) gear boxes
iv) small size electric motors.
v) rope sheaves, crane hooks, hoisting drums.



1) Sliding Contact Bearing.



2) Rolling Contact Bearing.

* Types of Rolling Contact Bearings.

For starting and moderate speeds,

frictional losses : Rolling Bearing is less than hydrodynamic Bearing.

Beacuse rolling Bearings results in less Coeff. of friction. Hence they are called "Antifriction" Bearings.

However there is always friction between rolling element and inner & outer cages.

Rolling Contact Bearing Consists of. parts.

- ① Inner and outer races
- ② Rolling element like Ball, roller, needle.
- ③ Cage — holds the rolling elements together and spaces them evenly around periphery of shaft.

Type of Rolling ~~Bearing~~ element

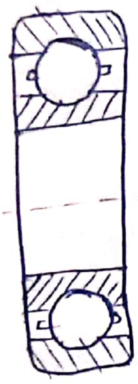
- Ball Bearings
- Cylinderical Roller Bearing
- Taper Roller Bearing
- needle Bearing.

Type of direction of load

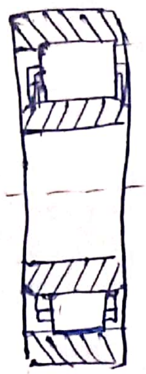
- Radial Bearing
- Thrust Bearing.

However no clear destinction between two types. Certain types of thrust Bearing can take radial loads. while some radial — " — " " thrust — " —

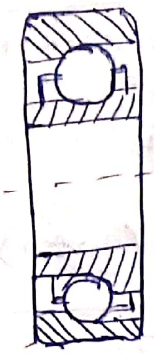
1) Deep groove Ball Bearings



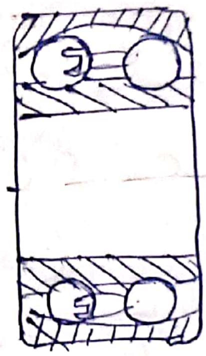
a) Deep groove



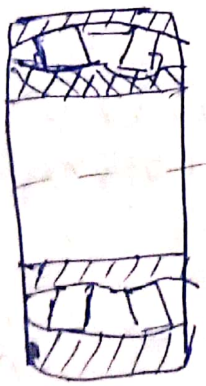
b) Cylindrical roller.



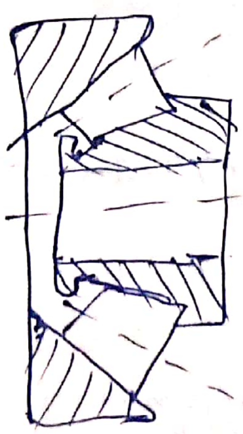
Angular contact.



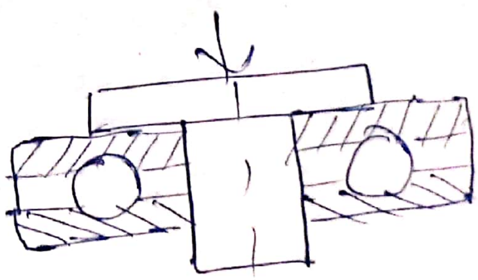
Self aligning



Spherical roller



Taper. Roller.



Thrust Ball Bearing

15.3 Problem

Single row deep groove ball bearing

$F_r = 3$ kN

$n = 600$ rpm

$L_{10h} = 30{,}000$ h

$d = 40$ mm

Select bearing.

$P = F_r = 3000$ N

$$L_{10} = \frac{60(n) L_{10h}}{10^6}$$

$$= \frac{(60)(600)(30000)}{10^6}$$

$$\boxed{L_{10} = 1080 \text{ million rev.}}$$

$$C = P (L_{10})^{1/p}$$

$$= (3000)(1080)^{1/3}$$

$$\boxed{C = 30779.57 \text{ N}}$$

From table,

for $d = 40$ mm and $C = 30779.57$ N

∴ The $C = 30700$ matches closely

~~Hence~~ and $30779.57 - 30700 = 79.57$ N

(It's negligible)

Hence for $C = 30700$ N

Designation = 6208 is selected

15.4

① Deep groove Ball Bearing :-

- most frequently used
- radius of ball < radius of groove
- there is point contact

Advantages

1) High load carrying capacity
2) takes both radial & axial loads
3) due to point contact
↓
friction less
↓
temp less.
↓
good performance in high speed.
4) less noise
5) available from few millimeter to 400 mm.

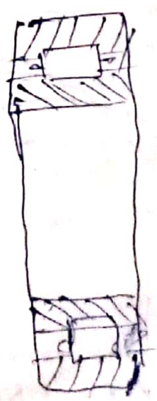
Disadvantages

① not self alligning
② Poor rigidity

due to point contact

② Cylindrical Roller Bearing.

- when more load carrying capacity required.
- line contact



Advantages :

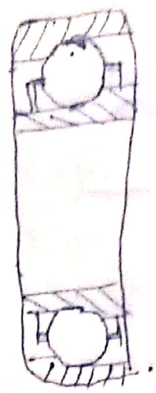
① high load carrying capacity
② more rigid than ball bearing
③ Coeff of friction is low
↓
less friction loss

Disadvantages:-

① Don't take thrust load.
② not self alligning.
③ more noise.

Scanned with CamScanner

3) Angular contact Bearing :-



— The grooves in inner & outter races are so shaped ~~that~~ that line of reaction at the contact between balls and races makes and angle with axis of Bearing.

— This reaction has two components
- radial
- axial

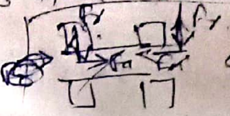
Advantages

① Can be used for both radial & thrust loads

② load carrying capacity is more than deep groove ball bearing.
as ~~there~~ one side of outer race is cut to insert more number of balls.

Disadvantages

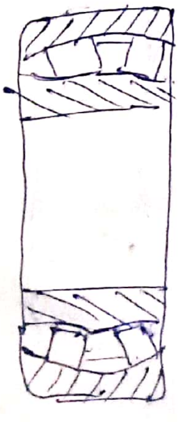
① Two bearings are required to take thrust load in both direction.



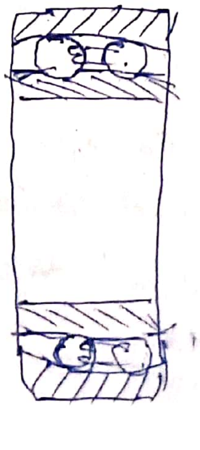
② no axial play allowed

③ requires initial pre loading.

4) Self - alligning Bearing



Spherical



Self alligning

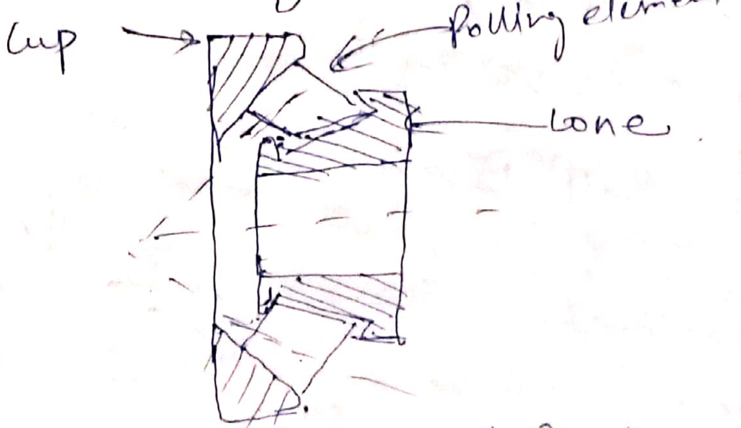
Two types

- Self alligning ball Bearing
- Spherical roller Bearing.

- It consist of two Balls, which roll on common Spherical Surface.
- The Shaft, Inner race and Balls with Cage can freely roll and adjust itself to misallignments.

- Self alligning Spherical roller bearing can take more radial and thrust load as compared to Self alligning ball bearing.
- minor missallignment can be allowed.
- used in agricultural machinery, ventilators, railway axle boxes.

5) <u>Taper roller Bearing</u>



- Rolling element is in form of frustum.
- axis of individual meet at common apex on axis of bearing. (For kinematic analysis this is important for pure rolling motion.
- The resultant reaction makes axis with Bearing.
- They take both radial and axial loads.

- If A taper bearing if subjected to thrust only than it will induce radial load also & vice versa. so we have to take pairs.

Advantages:-

1. Take radial & thrust loads
2. more rigidity
3. easy assemble and disassemble.

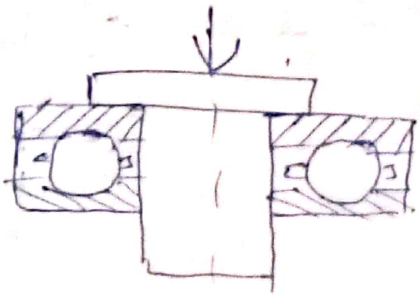
Disadvantages

1. Pairs must be used
2. The axial position of bearing is adjusted with preload to make common apex.
3. missallignment can't be tolerated
4. Costly.

used in cars and trucks. propeller shaft and differential, Rail-road axle-boxes. large size rolling mills.

6) Thrust Ball Bearing

- It consist of row of balls in two rings.
- shaft ring and housing ring.



- It carrying only axial loads in one direction
- If large balls used, high thrust load can be carried in less space ← major advantage

Disadvantage

1) Cannot take radial load
2) not self-alligining.
3) performance in Satisfactory at low & medium speed.
4) They can operate well on vertical shafts.
5) Continuous pressure should be applied by Spring to keep both rings together

used in worm gear boxes and crane hooks.

materials of Bearing Cotter

— Balls & Inner race, outer race
→ high Carbon steel (SAE 52100 or AISI 5210)

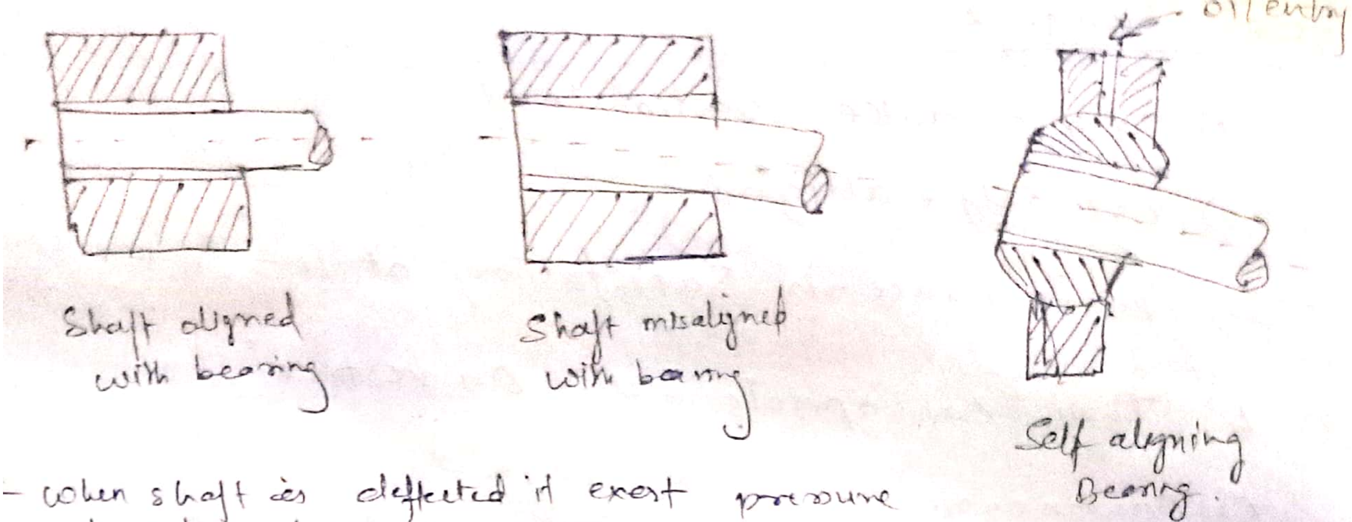
* Balls & races are through hardened to obtain a minimum hardness of 58 Rockwell C.

— Cages made from stampings of low carbon steel.

— rollers are made of case hardened steels (AISI 3310, 4620 or 8620)
they are Case carburised to obtain Surface Hardness of 58 Rockwell C.

* Principle of self-aligning Bearing:-

- The Self aligning bearing is required to tolerate small amount of misalignment between axis of Shaft and bearing.
- The misalignment may be due to deflection of the shaft under load or due to tolerances of individual component.



- when shaft is deflected it exert pressure at edges of bearing.
- The edge pressure is dangerous, may lead to wear of bearing and breakdown of oilfilm.
- In self-aligning bearing, external surface of bearing bush is made spherical.
- The centre of this spherical surface is at centre of bearing
- Therefore bush is free to roll in its seat align itself with the journal.
- This principle is used in self-aligning ball bearings and spherical roller bearings.
- Self aligning bearing are commonly employed when accurate alignment is impossible.

* Selection of Bearing type:-

Guidelines.

1) For low loads ~~and~~ medium loads :- ball bearings
   " high " & large ~~beam~~ dia shaft : roller bearing
2) when misalignment :- self aligning roller bearing and spherical roller bea.
3) Thrust Ball Bearing → medium thrust load.
   Cylindrical roller thrust Bearing → heavy thrust load.
4) Deep groove Ball bearings, Angular contact, spherical roller bearing suitable for radial ~~ther~~ and thrust - two component acting on bearing
5) Deep groove Ball Bearing, Angular contact, cylindrical → High speed.
6) when rigidity is criteria, Double row cylindrical or taper Bearing are used.
7) if noise is criteria than deep groove ball bearings are used

Static load Carrying Capacity :-

Static load is defined as the load acting on the bearing when shaft is stationary.

— It produces permanent deformation.

— Permissible static load depends upon permissible magnitude of permanent deformation.

— In practice a total deformation of 0.0001 of the ball or roller is tolerated

" The static load carrying capacity of a bearing is defined as the static load which corresponds to a total permanent deformation of balls and races at most heavily stressed point of contact equal to 0.0001 of the ball diameter ".

— The values of static load carrying capacities are directly given in manufacturer's Catalogue.

If noise and smoothness are not critical a higher deformation upto 4 times the static load carrying capacity may be permissible.

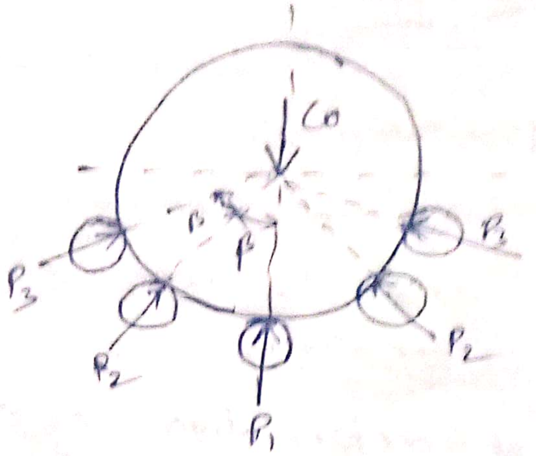
Stribeck's Equation :-

Stribeck's equation gives the static load capacity of bearing. It is based on following assumptions:-

1) Races are rigid and retain their circular shape.

2) The balls are equally spaced.

3) The balls in upper half do not support any load.



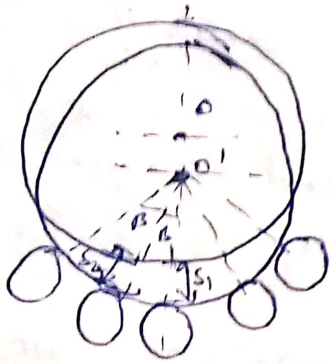
As

C_o = Static load

Fig:- Forces acting on inner races

Considering the equilib^m of forces in vertical direction.

$$C_o = P_1 + 2P_2 \cos\beta + 2P_3 \cos(2\beta) + \dots \quad \text{—— (a)}$$



As Races are rigid only Balls will deform.

δ_1 = deformation of Ball 1 at most heavily stressed condition.

Fig:- Deflection of Inner race

So δ_1 will also be the deflection of Inner race w.r.t Outer race.

So; δ_1, δ_2 - - - are radial deflection of the respective Balls.

$$\delta_2 = \delta_1 \cos\beta.$$

$$\boxed{\frac{\delta_2}{\delta_1} = \cos\beta} \quad \text{—— (1)}$$

$$\delta_3 = \delta_1 \cos 2\beta$$

$$\boxed{\frac{\delta_3}{\delta_1} = \cos 2\beta.}$$

According to ~~Hertz~~ Hertz's equtn.
relation between load & deflection is given by,

$$\delta \propto (P)^{2/3}.$$

Therefore

$$\delta_1 = C_1 P_1^{2/3}$$

and

$$\delta_2 = C_1 P_2^{2/3}$$

$$\boxed{\frac{\delta_2}{\delta_1} = \left(\frac{P_2}{P_1}\right)^{2/3}} \quad \text{--- (2)}$$

So, from equtn (1) & (2)

$$\left(\frac{P_2}{P_1}\right)^{2/3} = \cos\beta$$

$$\frac{P_2}{P_1} = (\cos\beta)^{3/2}$$

$$\boxed{P_2 = P_1 (\cos\beta)^{3/2}}$$

Similarly.

$$\boxed{P_3 = P_1 (\cos 2\beta)^{3/2}}.$$

and so on.

$$\delta_3 = \delta_1 \cos 2\beta$$

$$\frac{\delta_3}{\delta_1} = \cos 2\beta.$$

$$\left(\frac{P_3}{P_1}\right)^{2/3} = \frac{\delta_3}{\delta_1}$$

$$\left(\frac{P_3}{P_1}\right)^{2/3} = \cos 2\beta$$

So, substituting in equtn (a)

$$C_0 = P_1 + 2P_2 \cos\beta + 2P_3 \cos 2\beta + \cdots$$

$$C_0 = P_1 + 2P_1(\cos\beta)^{3/2}\cos\beta + 2P_1(\cos 2\beta)^{3/2}\cos 2\beta \cdots$$

$$C_0 = P_1\left[1 + 2(\cos\beta)^{3/2}\cos\beta + 2(\cos 2\beta)^{3/2}\cos 2\beta \cdots\right]$$

$$\frac{3}{2} + 1 = \frac{5}{2}$$

$$C_0 = P_1\left[1 + 2(\cos\beta)^{5/2} + 2(\cos 2\beta)^{5/2} \cdots\right]$$

$$\boxed{C_0 = P_1 M}$$

where

$$M = 1 + 2(\cos\beta)^{5/2} + 2(\cos 2\beta)^{5/2} \ldots$$

If z no. of ball are there

$$\beta = \frac{360}{z}$$

z	8	10	12	15
M	1.84	2.28	2.75	3.47
(z/m)	4.35	4.38	4.36	4.32

So $\frac{z}{m}$ value is nearly constant

So stribeck suggested the value for $\left(\frac{z}{m}\right)$ as 5.

So,

$$\boxed{M = \frac{z}{5}}$$

$$\boxed{C_0 = \frac{P_1 \cdot z}{5}}$$

From experimental evidence.

$$\boxed{P_1 = kd^2}$$

k = factor depending upon radius of curvature and E

d = Ball diameter.

$$\boxed{C_0 = \frac{kd^2 z}{5}}$$ — Stribeck's Equation.

* Dynamic Load Carrying Capacity -

"The dynamic load carrying capacity of the bearing is based on fatigue life of bearing."

"The life of an individual ball bearing is defined as the number of revolutions (or hours of service at some given constant speed), which the bearing runs before the first evidence of fatigue crack in balls or races."

Bearings are rated on two criteria

- Average life of a group of bearing
- life, which 90% of the bearing ~~with~~ will reach or exceed. * This is widely used.

"The rating life of a group of apparently identical ball bearings is defined as the number of revolutions that 90% of the bearings will complete or exceed before the first evidence of fatigue crack."

Terms used for rating life :- minimum life, catalogue life, L_{10} life or B_{10} life

The life of an individual ball bearing may be different from rating life.

~~Statistic~~ statistically, it can be proved that the life, which 50% of a group of bearings will complete or exceed is approximately five times the rating or L_{10} life.

means majority of bearing will have actual life more than rated

"The dynamic ~~lao~~ load carrying capacity of a bearing is defined as the radial load in radial bearings (or thrust load in thrust bearings) that can be carried for a minimum life of one million revolutions."

The dynamic load carrying capacity is based on the assumption that the inner race is rotating while outer is stationary.

* Equivalent Bearing Load

In actual applications, the force acting on the bearing has two components

- ① radial
- ② Thrust.

It is therefore necessary to convert the two components acting on bearing into a single hypothetical applied to dynamic load fulfilling the conditions load carrying capacity

"The equivalent dynamic load is defined as the constant radial load in radial bearings (or thrust load in thrust bearings), which if applied to the bearing would give same life as that which the bearing will attain under actual conditions of forces".

Expression for equivalent dynamic load,

$$\boxed{P = XVF_r + YF_a}$$

P = equivalent dynamic load (N)

F_r = radial load

F_a = axial load.

V = race - rotation factor

If Inner race rotating and outer stationary ; V = 1

" outer " " " Inner " , V = 1.2

Most of cases , V = 1

so,

$$\boxed{P = X F_r + Y F_a}$$

when bearing subjected to pure radial load,

$$\boxed{P = F_r}$$

when bearing subjected to pure thrust load

$$\boxed{P = F_a}$$

* load life Relationship

$$\boxed{L_{10} = \left(\frac{C}{P}\right)^{p}}$$ ← bearing life

L_{10} = rated bearing life (million rev)

C = Dynamic load Capacity

$p = 3$ (for ball bearings)

$p = \frac{10}{3}$ (for roller bearings)

$$(L_{10})^{1/p} = \left(\frac{C}{P}\right)$$

$$\boxed{C = P \cdot (L_{10})^{1/p}}$$

$$L_{10} = \frac{60 n L_{10h}}{10^6}$$

L_{10h} = rate bearing life (in hours)

n = speed of rotation (rpm)

15.1 Problem

$F_r = 5 kN$

$L_{10h} = 8000 h$

$C = ?$

$n = 1450$ rpm

$$L_{10} = \frac{60 n L_{10h}}{10^6}$$

$$L_{10} = \frac{60(1450)(8000)}{10^6}$$

$$\boxed{L_{10} = 696 \text{ million rev.}}$$

Dynamic load carrying capacity (C)

$$C = P(L_{10})^{1/p}$$

$$= F_r (L_{10})^{1/p}$$ — only F_r is acting

$$= (5 \times 10^3)(696)^{1/3}$$

$$\boxed{C = 44310.48 N}$$

15.2 Problem

$C = 26 kN$

$L_{10h} = 8000 h.$

$n = 300$ rpm.

$F_r = ?$

$$L_{10} = \frac{60 n L_{10h}}{10^6}$$

$$= \frac{(60)(300)(8000)}{10^6}$$

$$\boxed{L_{10} = 144 \text{ million rev.}}$$

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$$C = P(L_{10})^{1/p}$$

$$26 \times 10^3 = F_R(144)^{10/3}$$

$$\boxed{F_R = 5854.16N}$$

* Selection of Bearing Life :-

- The selection of proper size of bearing is based on the life expectancy of bearing.
- Based on past experience the bearing life are.

Fig : Bearing life for wheel Applications. (where speed of rotation is not const)

Sr. no	wheel application	Life. (million rev.)
1.	Automobile Cars.	50
2	Trucks.	100
3	Trolley Cars.	500
4	Rail road cars.	1000

Table :- Bearing life for Industrial application (rotation is const and expressed in hours service)

(i) m/c used intermittently such as lifting tackle, hand tools & household appliances — 4000 - 8000h.

ii) m/c used for eight hours of service per day, such as electric motor & gear drives — 12000 - 20000h.

iii) m/c's used for continous operations (24hr per day) such as pumps, compressor & conveyor. — 40000 - 60000h.

* Load factor

- Forces acting on the bearing are calculated by considering the equlbm of forces in vertical and horizontal ~~p~~ planes

- These elementary equations do not take into consideration the effect of dynamic loads.

- These dynamic load carrying capacity is determined by multiplying elementary eqution with ie "load factor".

- In gear drives there is additional dynamic load due to inaccuracy in tooth profile & elastic deformation of teeth.

- In chains & belts it is due to vibration.

	Type of drive	load factor
(A)	Gear drives	
	i) Rotating m/c free from impact like electric motor and turbo - compressor	1.2 - 1.4
	ii) Reciprocating m/c like I.C. Engine and compressor	1.4 - 1.7
	iii) Impact m/c like hammer mills	2.5 - 3.5
B)	Belt drives	
	1) V - Belt	2.0
	2) Single ply leather Belt	3.0
	3) Double - ply leather Belt	3.5
(C)	Chain drives	1.5

* Selection of Bearing from manufacturer's Catalogue :-

Steps :-

1) Calculate F_r and F_a, and determine diameter of shaft, where bearing is to fitted.

2) Select the ~~bea~~ type of bearing depending on the application

3) Determine value of X and Y, and Radial and Thrust factor } from manufacturer Catalogue

→ In table the values are given.

The values depend upon $\left(\frac{F_a}{F_r}\right)$ and $\left(\frac{F_a}{C_o}\right)$

C_o = static load capacity.

The selection of bearing is done on trial and error.

→ ~~To begin~~ The ~~selection~~ Static and dynamic load for single row deep groove ball bearing are shown in table.

The value of C_o can be find from ratio $\left(\frac{F_a}{C_o}\right)$ and $\left(\frac{F_a}{F_r}\right)$.

4) Calculate dynamic ~~d.~~ load from equation.

$$P = XF_r + YF_a.$$

5) make decision on expected L_{10h} and express it in L_{10}

6) calculate $C = P(L_{10})^{1/p}$

7) check from the bearing series whether bearing has the required dynamic capacity.

If not select bearing of next series and go back to Step 3.

Example

Select Single row deep groove ball Bearing.

$d = 75 mm$

$n = 125 rpm$

$F_r = 21 kN$

$F_a = 0$

$L_{10h} = 10,000$ hrs.

$$L_{10} = \frac{60n(L_{10h})}{10^6}$$

$$= \frac{(60)(125)(10,000)}{10^6}$$

$$\boxed{L_{10} = 75 \text{ million rev.}}$$

∴ since

$$P = F_r = 21 \times 10^3 N$$

$$C = P (L_{10})^{1/p}$$

$$= 21 \times 10^3 (75)^{1/3}$$

$$\boxed{C = 88560.43 N.}$$

for $d = 75 mm$ see table which shows.

No. 6015 (C = 39700 N)

No 6215 (C = 66300 N)

No. 6315 (C = 112000 N) ← Selected

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The term "series" is sometime used in manufacturer's Catalogue.

Series (For a given dia of shaft)

Light series
- (The dimensions of ball and races are smaller)
- load carrying capacity is less
- Cost is less

medium series
(They 30 to 40% higher load carrying capacity light series)

goes on increasing with series.

Heavy series.
- dynamic load carrying capacity is more
- Cost is more
- They 20 to 30% more load carrying capacity than medium series

Trial error method starts with light series. thean medium and then atlast Heavy.

Designation of Bearing

Rolling Contact Bearing is usually designated by four digits.

1) Last two digit :- Indicate Bore diameter of Bearing in mm.
(i.e Bore dia. divided by 5)

For e.g :- XX07 means a bearing of 35 mm ~~dia~~ bore dia.

2) 3rd digit from right :- Indicates series of Bearing

1 - Extra light Series

2 - Light series

3 - medium series

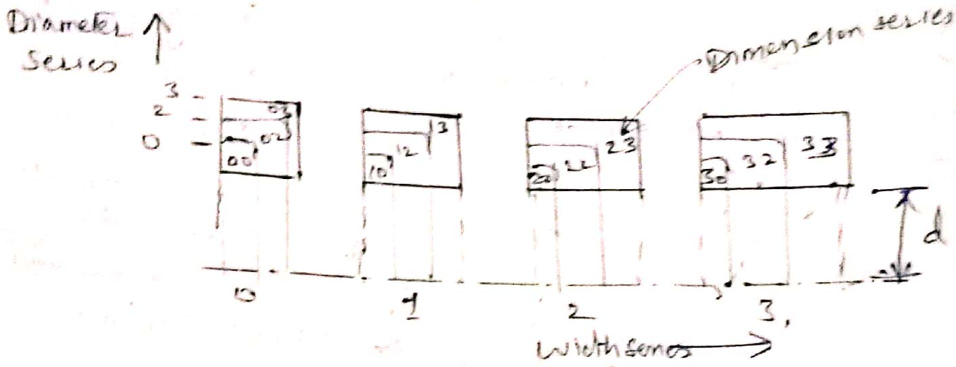
4 - Heavy series.

for e.g. ~~XXXXX~~ X307 means medium series Bearing with 35 mm dia.

3) The Fourth digit & Sometime 5th digit :- type of rolling Contact Bearing

For. e.g. 6 → deep groove Ball Bearing

* ISO Plan for the dimension series of Bearing.



Dimension Series is Two digits

AB

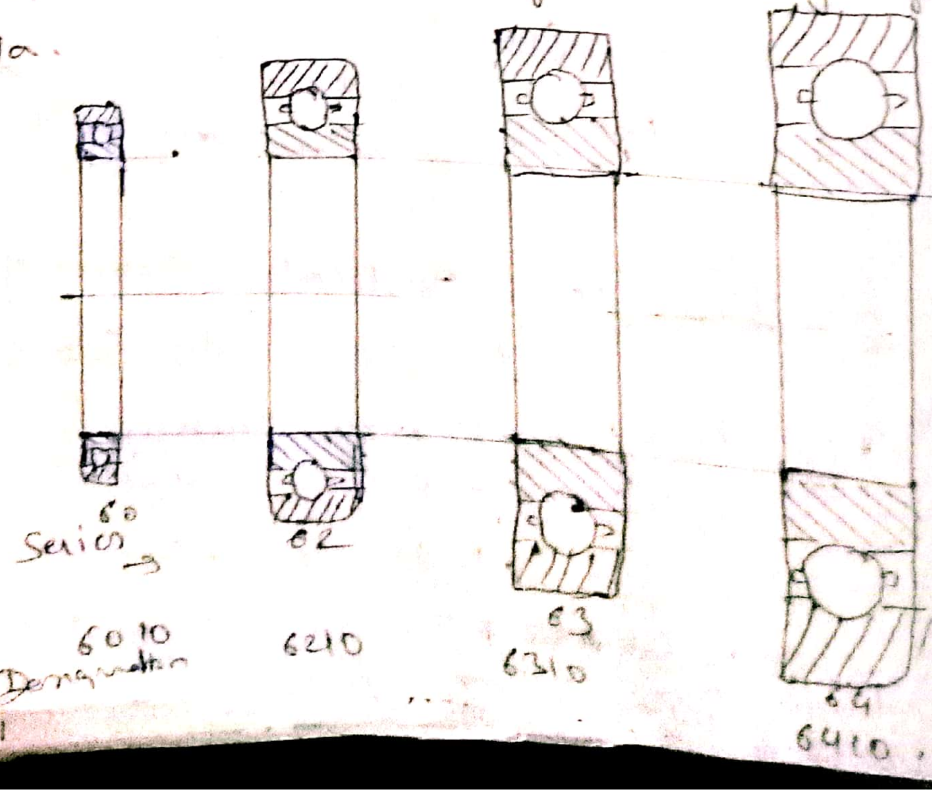
A → width series

B → diameter series

It is in increasing width.
8, 0, 1, 2, 3, 4, 5, 6

It is diameter series
7, 8, 9, 0, 1, 2, 3, 4 in order of ascending order of O.D.

⇒ Proportional dimension of SKF Bearing for 50mm dia.



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15.4 Problem

Single row deep groove bearing

$F_r = 8000N$

$F_a = 3000N$

$n = 1200$ rpm

$L_{10h} = 20{,}000h$

$d = 75mm.$

Ans

$\therefore P = XF_r + YF_a$

In order to find X and Y refer table.

$$\frac{F_a}{F_r} = \frac{3000}{8000} = 0.375$$

By Trial & error let us consider.

$$\left(\frac{F_a}{F_r}\right) > e$$

In this case Y varies from 1.0 to 2.0

Hence taking average value as 1.5.

Therefore,

$X = 0.56$, $Y = 1.5$:

so,

$$P = (0.56)(8000) + (1.5)(3000)$$

$$\boxed{P = 8980N}$$

$$L_{10} = \frac{60 n L_{10h}}{10^6}$$

$$= \frac{(60)(1200)(20{,}000)}{10^6}$$

$$\boxed{L_{10} = 1440 \text{ million rev.}}$$

$$C = P(L_{10})^{1/p}$$

$$= (8980)(1440)^{1/3}$$

$$\boxed{C = 101406.04N}$$

From table,

for $d = 75\,mm$ and $C = 101406.04\,N$

So we select $C = 112000\,N$ i.e 6315 Designation

and $C_o = 72000\,N$

So,

$$\frac{F_a}{F_r} = 0.375$$

and

$$\frac{F_a}{C_o} = \frac{3000}{72000} = 0.04167$$

So for above value of $\frac{F_a}{C_o} = 0.04167$ the

$e = 0.24$. (approximately)

$$\therefore \frac{F_a}{F_r} > e$$

So from table, for $\left(\frac{F_a}{C_o}\right) = 0.04167$

$X = 0.56$ But $Y = ?$.

$\left(\frac{F_a}{C_o}\right)$	$\left(\frac{F_a}{F_r}\right) \leq e$		$(F_a/F_r) > e$		e
	X	Y	X	Y	
			0.56	2.0	
0.025			0.56	1.8	
0.040			0.56	?	
0.04167			0.56	1.6	
0.070					

So By interpolation

$$\frac{0.070 - 0.040}{0.04167 - 0.040} = \frac{1.6 - 1.8}{Y - 1.8}$$

$$\boxed{Y = 1.79}$$

So, $P = XF_r + YF_a$

$= (0.56)(8000) + (1.79)(3000)$

$$\boxed{P = 9850 N}$$

$$C = P(L_{10})^{1/p}$$

$$= 9850(1440)^{1/3}$$

$$\boxed{C = 111230.46 N}$$

So, from table
for $d = 75 mm$

$C = 112000$ is close

So we select 6315

15.5
Problem

$n = 720 rpm$

$P_1 = 498 N$

$P_2 = 166 N$

$P_t = 497 N$

$P_r = 181 N$

$W = 100 N$

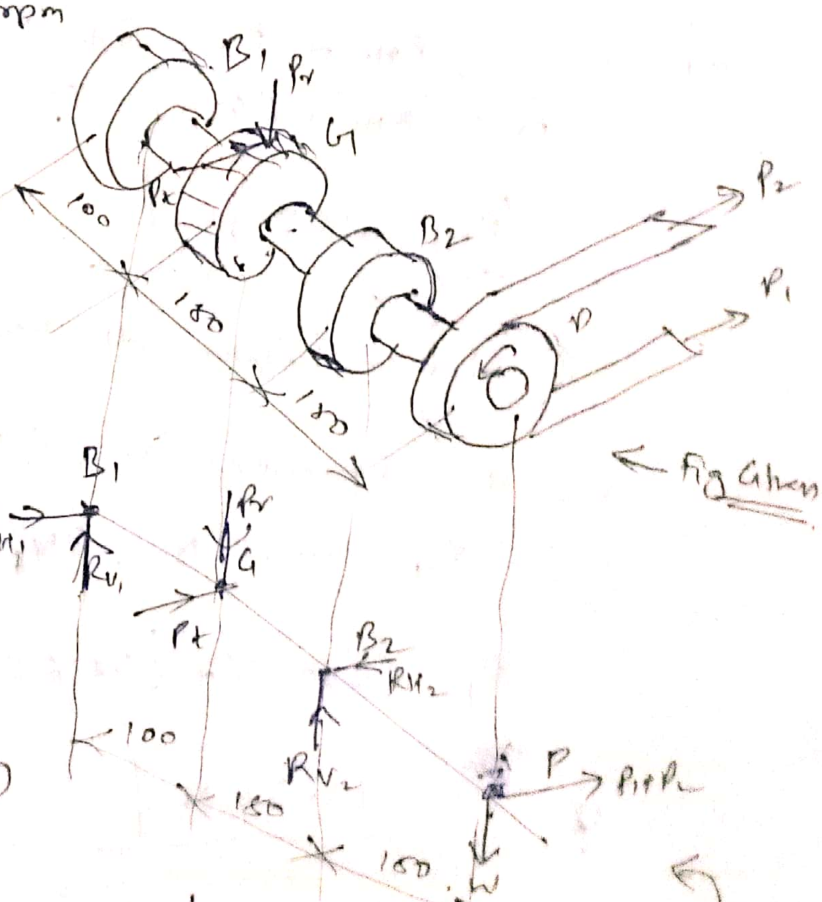
$d_1 = 10 mm$

$d_2 = 20 mm$

load factor = 2.5

$L_{10h} = 8000 h$

Select Bearings.
(deep groove ball bearing)



Considering equlb^m of force as shown in fig
1) vertical plane.

$$\Sigma M_{B1} = 0 = P_r \cdot (100) - (R_{V2} \times 280) + W(400)$$

$$0 = (181)(100) - (R_{V2} \times 280) + (100 \times 400)$$

$$\boxed{R_{V2} = 232.4 N.}$$

$\Sigma F_y = 0 = R_{V_1} - P_r + R_{V_2} - W$

$0 = R_{V_1} - 181 + 232.4 - 100$

$\boxed{R_{V_1} = 48.6\,N.}$

2) Horizontal plane :-

$\Sigma M_{B_1} = 0 = (P_t \cdot 100) - (R_{H_2} \cdot 250) + (P_1 + P_2) \cdot 400$

$0 = (497 \times 100) - (R_{H_2} \times 250) + [(498 + 166) \times 400]$

$\boxed{R_{H_2} = 1261.2\,N}$

$\Sigma F_x = 0 = R_{H_1} + P_t - R_{H_2} + (P_1 + P_2)$

$0 = R_{H_1} + (497) - (1261.2) + (498 + 166)$

$\boxed{R_{H_1} = 100.2}$

Hence Assume direction is Correct.

So, the Reactions on Bearing B_1

$R_{H_1} = 100.2\,N$, $R_{V_1} = 48.6\,N$

$R_1 = \sqrt{(100.2)^2 + (48.6)^2}$

$\boxed{R_1 = 111.36\,N.}$

Reaction on Bearing B_2

$R_{H_2} = 1261.2\,N$ and $R_{V_2} = 232.4\,N.$

$R_2 = \sqrt{(1261.2)^2 + (232.4)^2}$

$\boxed{R_2 = 1282.43\,N}$

So,

$F_{r_1} = 111.36 N = P_1$

and

$F_{r_2} = 1282.43 N = P_2$

$$L_{10} = \frac{60(n) L_{10h}}{10^6}$$

$$= \frac{(60)(720)(8000)}{10^6}$$

$$\boxed{L_{10} = 345.6 \text{ million rev.}}$$

for Bearing 1,

$$C_1 = P_1 (L_{10})^{1/p} (\text{load factor})$$

$$= (111.36)(345.6)^{1/3}(2.5)$$

$$\boxed{C_1 = 1953.71 N}$$

so from table

for diameter = 10mm & $C_1 = 1953.71N$

So Bearing designation 6000 which has $C = 4,620N$ is selected

for Bearing 2

$$C_2 = P_2 (L_{10})^{1/p} (\text{load factor})$$

$$= (1282.42)(345.6)^{1/3}(2.5)$$

$$\boxed{C_2 = 22499.09N}$$

So from table $d_2 = 20mm$, $C_2 = 22499N$.

designation 6404 is selected.

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Designation : 6002

F_a = 1000N

F_r = 2200N

L_{50} = ?

Ans

L_{50} = 540 ← we know this.

∴ Refering table

Designation 6002 is used for

d = 15mm

C = 5590 N

C_0 = 2500 N

$$\frac{F_a}{F_r} = \frac{1000}{2200}$$

$$\boxed{\frac{F_a}{F_r} = 0.455}$$

$$\frac{F_a}{C_0} = \frac{1000}{2500}$$

$$\boxed{\frac{F_a}{C_0} = 0.400}$$

Since when we see table

$\left(\frac{F_a}{C_0}\right)$	$\left(\frac{F_a}{F_r}\right) \leq e$		$\left(\frac{F_a}{F_r}\right) > e$		e
	X	Y	X	Y	
0.250	1	0	0.56	1.2	0.37
0.400	1	0	0.56	?	?
0.500	1	0	0.56	1.0	0.44

← (0.37 to 0.44)

But we can see that e value will be in between 0.37 and 0.44, for $\frac{F_a}{C_0}$ = 0.400 so we can say that

$$\frac{F_a}{F_r} > e.$$

So we will refer 2nd part of table.

$$\frac{0.400 - 0.25}{0.500 - 0.25} = \frac{y - 1.2}{1.0 - 1.2}$$

$$\boxed{Y = 1.08}$$

and $X = 0.56$.

So,

$$P = X F_r + Y F_a$$

$$= (0.56)(2200) + (1.08)(1000)$$

$$\boxed{P = 2312 N}$$

$$C = P (L_{10})^{1/b}$$

$$5590 = (2312)(L_{10})^{1/3}$$

$$\boxed{L_{10} = 14.13 \text{ mill rev.}}$$

$$\therefore L_{50} = 5 L_{10}$$

$$= 5 \times 14.13$$

$$\boxed{L_{50} = 70.67 \text{ million rev.}}$$